

تابع خطی $f(x) = ax + b$ $\rightarrow x = 1 \rightarrow 2f(1) = 3 + f(1) \rightarrow f(1) = 3$

تابع خطی $f(2x+1) + f(x+1) = 3x+3$ $\rightarrow a(2x+1) + b + a(x+1) + b = 3ax + 3a + 2b = 3x + 3$

$\rightarrow a = 1, b = 0 \rightarrow f(x) = 3x$ $\rightarrow y = \sqrt{f(x) - x^2} = \sqrt{3x - x^2} \rightarrow x_0 = -\frac{b}{2a} = \frac{3}{2}$

$y_{\max} = \frac{9}{4} \rightarrow R = (-\infty, \frac{9}{4}]$ $\xrightarrow{\text{رادیكال}} R = \sqrt{(-\infty, \frac{9}{4}]} = [0, \frac{9}{4}]$ جواب آخر

سادگی کردن ضابطه $f(x) = \frac{x(x+2)(x-2)}{x+2} \xrightarrow{x \neq -2} = x^2 - 2x$ $\rightarrow x_0 = -\frac{b}{2a} = 1 \rightarrow y_0 = -1$

$x = -2, 4$ غشوق است

$R = [c, +\infty) - \{1\}$ $\rightarrow D = R - \{a, b\} = \{-2, 4\} - \{a, b\}$ جواب آخر

$c = -1 \rightarrow f(\frac{4+(-1)}{-2}) = f(-1) = (-1)^2 - 2(-1) = 1 + 2 = 3$

از آنجایی که قدری رادیكال فرد بوده (۳) و عبارت تابعی خطی است عبارت زیر رادیكال از مرتبه فرد (۳) باشد:

$y = \sqrt[3]{x^3 + ax^2 + bx + c - 1} = \sqrt[3]{x^3 - 6x^2 + 12x - 1} = \sqrt[3]{(x-2)^3} = x-2$ $\rightarrow a = -6, b = 12$

$y_{\min} = x_{\min} - 2 = -6 - 2 = -8$ $\rightarrow y_{\max} = x_{\max} - 2 = 12 - 2 = 10$ $\rightarrow R = [-8, 10]$ ✓

$y_1 = (\frac{x+1}{1-x})^{\frac{1}{2}} \rightarrow \frac{x+1}{1-x} > 0 \rightarrow D_{y_1} = (-1, 1) = R_{y_1}$

$y_2 = \frac{ax^2 - 1}{x^2 + b} \xrightarrow{x \rightarrow \infty} y = a \rightarrow R = [-\frac{1}{b}, a) \rightarrow R = (-\frac{1}{b}, a) \rightarrow a = b = 1$

$x = 0 \rightarrow y = -\frac{1}{b}$ چون در این سرریشه می شود پس ریشه خارج است تا باز بماند

$a + b = 2$

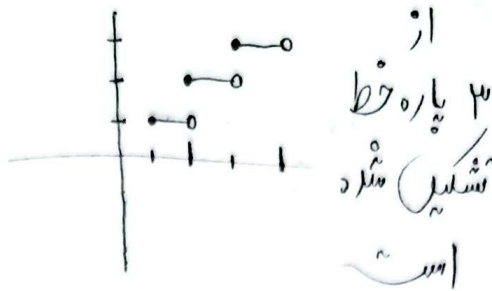
$\rightarrow y = \frac{|x| + [x]}{x} \quad (-1 < x < 1, x \neq 0)$

$\begin{cases} -\frac{(x+1)}{x} & -1 < x < 0 \\ 1 & 0 < x < 1 \\ 2 & x = 1 \end{cases}$

$x = -1$ $-1 < x < 0$ $0 < x < 1$ $x = 1$

$(-1, 0)$ $(1, 2)$

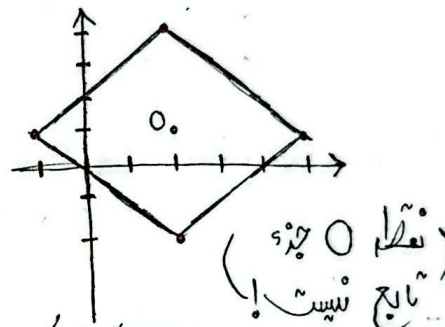
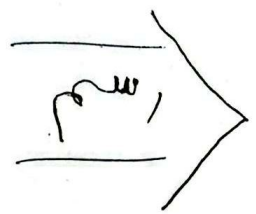
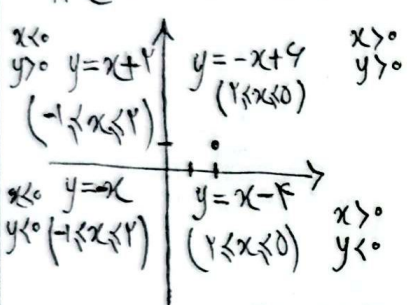
$$y = [x] + \left[x + \frac{1}{r} \right] \begin{cases} 1 & \frac{1}{r} < x < 1 \\ 2 & 1 < x < \frac{r}{r} \\ 3 & \frac{r}{r} < x < 2 \end{cases}$$



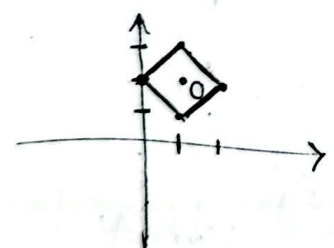
$$\begin{aligned} \rightarrow \left| \frac{x+1}{ax+r} \right| < 1 &\rightarrow -1 < \frac{x+1}{ax+r} < 1 \rightarrow \frac{(1+a)x+r}{ax+r} < 1 \\ &\rightarrow \frac{(1-a)x-r}{ax+r} < 0 \quad a=1 \\ \Rightarrow \frac{2x+r}{x+r} &\rightarrow \frac{-r}{x+r} < 0 \quad (-\infty, -r) \cup (-r, +\infty) \\ \Rightarrow \frac{-r}{x+r} &\rightarrow \frac{-r}{x+r} > 0 \quad (-r, +\infty) \end{aligned}$$

$D = (-r, +\infty) \rightarrow a-b = 1 - (-r) = 1+r$

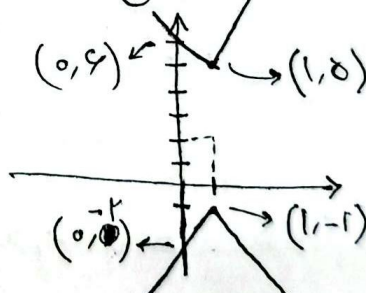
با توجه به شرایط و مرکز مربع



الف) تابع مربع: $|x-1| + |y-2| = 1$



تابع پیکان (ب)



$$y = 2x + |ax+b| \begin{cases} (r+a)x+b & x > -\frac{b}{a} \\ (r-a)x-b & x < -\frac{b}{a} \end{cases} \quad \begin{aligned} x=0 &\rightarrow b=-b \rightarrow b=0 \\ (a+r)x & \quad x > 0 \\ (-a+r)x & \quad x < 0 \end{aligned}$$

برای اینکه به تابع R باشد باید شرایط علامت باشد

$$\begin{aligned} a+r > 0 &\rightarrow a > -r \quad \text{و} \quad a+r < 0 \rightarrow a < -r \\ \Rightarrow -a+r > r &\rightarrow a < 0 \quad \text{و} \quad -a+r < 0 \rightarrow r < a \end{aligned}$$

$a \in (-r, r) \cup \emptyset \rightarrow a \in (-r, r)$