

تابع هائراس است $f(2(1)+1) + f(1)+2 = 3(1) + f(3) \rightarrow f(3)=3 \rightarrow f(x)=x$

$$y = \sqrt{f(x)-x^2} = \sqrt{-x^2+x} \quad -x^2+x \geq 0 \quad \left| \begin{array}{l} -\frac{b}{2a} = \frac{1}{2} \\ \frac{1}{4} \text{ max} \end{array} \right.$$

$$\Rightarrow R = \sqrt{(-\infty, \frac{1}{2}]} \Rightarrow R = [0, \frac{1}{2}]$$

$$f(x) = \frac{x^2-1}{x+2} = \frac{x(x-1)(x+2)}{(x+2)} \stackrel{x \neq -2}{=} x(x-1) = x^2-x \quad \left| \begin{array}{l} -\frac{b}{2a} = \frac{1}{2} \\ -1 \text{ min} \end{array} \right.$$

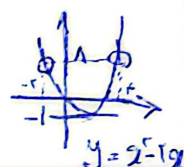
$$x^2-x \neq 1 \rightarrow x^2-x-1 \neq 0 \rightarrow (x-1)(x+2) \neq 0 \rightarrow x \neq 1, x \neq -2$$

$$\rightarrow D_f = \mathbb{R} - \{-2, 1\}$$

$$f\left(\frac{a+b}{2}\right) = f\left(\frac{-2+1}{2}\right)$$

$$\Rightarrow R_f = [-1, +\infty) - \{1\}$$

$$= f(-1) = (-1)^2 - (-1) = 2$$



$$y = \sqrt[3]{2^2+4x+b} = \sqrt[3]{(x-1)^3} = \sqrt[3]{x^3-4x^2+11x-1} \rightarrow \begin{cases} a = -4 \\ b = 12 \end{cases}$$

$$\rightarrow D_f = [-4, 12] \xrightarrow{y=x-1} R_f = [-1, 1]$$

$$y_1 = \left(\frac{x+1}{1-x}\right)^{\frac{1}{2}} \rightarrow \frac{x+1}{1-x} \geq 0 \rightarrow \frac{-1}{-1+1} \rightarrow D_{y_1} = [-1, 1]$$

$$y_r = \frac{ax^2-1}{x^2+b} \rightarrow \lim_{x \rightarrow +\infty} \frac{ax^2-1}{x^2+b} = a = 1$$

$$\Rightarrow a+b = 1+1 = 2$$

$$R_{y_r} = \left[-\frac{1}{b}, 1\right)$$

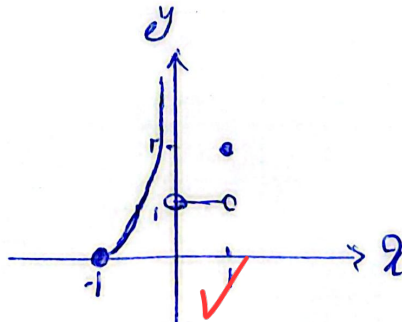
$$x=0 \rightarrow y_r = \frac{-1}{b} = -1 \rightarrow b=1$$

$$y = \frac{[x]}{x} + \frac{|x|}{x}$$

$$-1 < x < 0 \rightarrow y = \frac{-1}{x} + \frac{-x}{x} = -\frac{1}{x} - 1$$

$$0 < x < 1 \rightarrow y = 0 + \frac{x}{x} = 1$$

$$x=1 \rightarrow y = \frac{1}{1} + \frac{1}{1} = 2$$

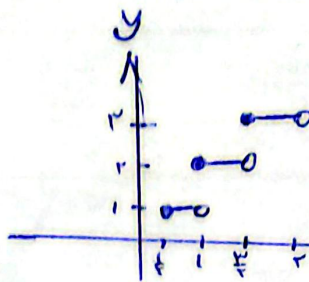


$$y = [x] + [x + \frac{1}{2}]$$

$$\frac{1}{2} \leq x < 1 \rightarrow y = 0 + 1 = 1$$

$$1 \leq x < 1.5 \rightarrow y = 1 + 1 = 2$$

$$1.5 \leq x < 2 \rightarrow y = 1 + 2 = 3$$



از ۳ نقطه شیب ۱ میسر است

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$$\left| \frac{x+1}{ax+3} \right| < 1 \rightarrow -1 < \frac{x+1}{ax+3} < 1 \rightarrow \textcircled{1} \quad \frac{x+1}{ax+3} < 1 \xrightarrow{ax+3 > 0} x+1 < ax+3 \rightarrow x(1-a) < 2 \rightarrow x < \frac{2}{1-a}$$

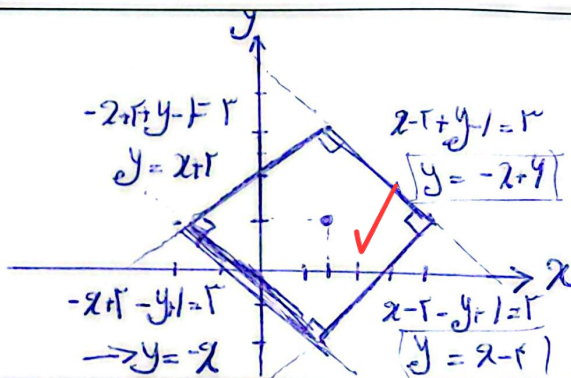
$$\rightarrow \textcircled{2} \quad \frac{x+1}{ax+3} > -1 \xrightarrow{ax+3 > 0} x+1 > -ax-3 \rightarrow x > \frac{-4}{1+a} \quad \textcircled{1}, \textcircled{2} \Rightarrow \frac{-4}{1+a} < x < \frac{2}{1-a}$$

$$\rightarrow \frac{2}{1-a} > \frac{-4}{1+a} \rightarrow \frac{2}{1-a} = \frac{-4}{1+a} \rightarrow 1+a = -2(1-a) \rightarrow 1+a = -2+2a \rightarrow a=3$$

$$x > \frac{-4}{1+a} = \frac{-4}{1+3} = -1 \rightarrow (b, \infty) = (-1, \infty) \Rightarrow a-b = 1-(-1) = 2$$

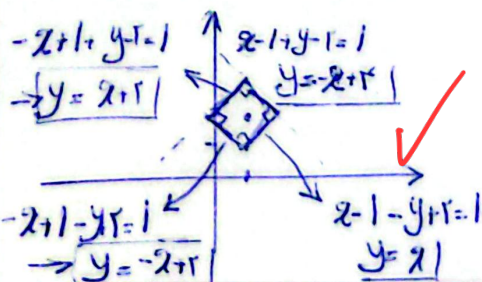
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$$|x-2| + |y-1| = 3$$



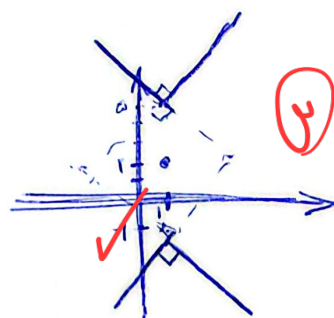
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$$|x-1| + |y-2| = 1$$



$$|x-1| - |y-2| = -3$$

$$\rightarrow |y-2| - |x-1| = 3$$



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$$y = rx + |ax+b|$$

$$R_f = \mathbb{R}$$

$$\text{if } ax+b \geq 0 \rightarrow rx+ax+b = (a+r)x+b$$

$$\text{if } ax+b < 0 \rightarrow rx-ax-b = (r-a)x-b$$

$$\rightarrow \frac{a+r}{r-a}x$$

$$\Rightarrow -a \leq r < 0 \rightarrow a \geq r \rightarrow \begin{cases} a \geq r \\ a \leq -r \end{cases}$$

$$-r < a < r$$

شیب قائم‌الزاویه
 $m_1, m_2 \neq 0$

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