

تابع هائراس است  $f(2(1)+1) + f(1)+2 = 3(1) + f(3) \rightarrow f(3)=3 \rightarrow f(x)=x$

$$y = \sqrt{f(x)-x^2} = \sqrt{-x^2+x} \quad -x^2+x \geq 0 \quad \left| \begin{array}{l} -\frac{b}{2a} = \frac{1}{2} \\ \frac{1}{4} \text{ max} \end{array} \right.$$

$$\Rightarrow R = \sqrt{(-\infty, \frac{1}{4}]} \Rightarrow R = [0, \frac{1}{2}]$$

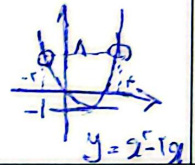
$$f(x) = \frac{x^2-1}{x+2} = \frac{x(x-1)(x+2)}{(x+2)} \stackrel{x \neq -2}{=} x(x-1) = x^2-x \quad \left| \begin{array}{l} -\frac{b}{2a} = \frac{1}{2} \\ -1 \text{ min} \end{array} \right.$$

$$x^2-x \neq 1 \rightarrow x^2-x-1 \neq 0 \rightarrow (x-1)(x+2) \neq 0 \rightarrow x \neq 1, x \neq -2$$

$$\rightarrow D_f = IR - \{-2, 1\}$$

$$f\left(\frac{a+b}{2}\right) = f\left(\frac{-2+1}{2}\right)$$

$$= f(-1) = (-1)^2 - (-1) = 2$$



$$\Rightarrow R_f = [1, +\infty) - \{1\}$$

$$y = \sqrt[3]{2^2+4x^2+bx-1} = \sqrt[3]{(x-1)^3} = \sqrt[3]{x^3-4x^2+11x-1} \rightarrow \begin{cases} a = -4 \\ b = 12 \end{cases}$$

$$\rightarrow D_f = [-4, 12] \xrightarrow{y=x^2-1} R_f = [-1, 1]$$

$$y_1 = \left(\frac{x+1}{1-x}\right)^{\frac{1}{2}} \rightarrow \frac{x+1}{1-x} \geq 0 \rightarrow \frac{-1}{-1+1} \rightarrow D_{y_1} = [-1, 1]$$

$$y_r = \frac{ax^2-1}{x^2+b} \rightarrow \lim_{x \rightarrow +\infty} \frac{ax^2-1}{x^2+b} = a = 1 \Rightarrow a+b = 1+1 = 2$$

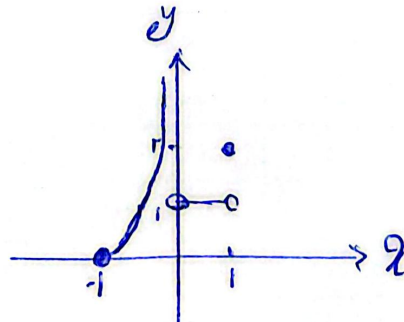
$$D_{y_r} = \left[-1, 1\right) \quad x=0 \rightarrow y_r = \frac{-1}{b} = -1 \rightarrow b=1$$

$$y = \frac{[x]}{x} + \frac{|x|}{x}$$

$$-1 < x < 0 \rightarrow y = \frac{-1}{x} + \frac{-x}{x} = -\frac{1}{x} - 1$$

$$0 < x < 1 \rightarrow y = 0 + \frac{x}{x} = 1$$

$$x=1 \rightarrow y = \frac{1}{1} + \frac{1}{1} = 2$$

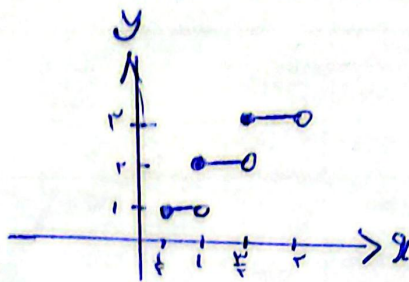


$$y = [x] + \left[x + \frac{1}{r}\right]$$

$$\frac{1}{r} \leq x < 1 \rightarrow y = 0 + 1 = 1$$

$$1 \leq x < 1 + \frac{1}{r} \rightarrow y = 1 + 1 = 2$$

$$1 + \frac{1}{r} \leq x < 2 \rightarrow y = 1 + 2 = 3$$



از ۳ نقطه شیب و شیب است

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$$\left| \frac{x+1}{ax+r} \right| < 1 \rightarrow -1 < \frac{x+1}{ax+r} < 1 \rightarrow \textcircled{1} \quad \frac{x+1}{ax+r} < 1 \xrightarrow{a>0} x+1 < ax+r \rightarrow x(1-a) < r \rightarrow x < \frac{r}{1-a}$$

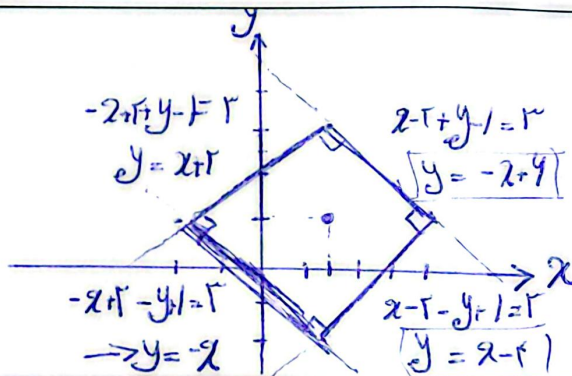
$$\rightarrow \textcircled{2} \quad \frac{x+1}{ax+r} > -1 \xrightarrow{a>0} x+1 > -ax-r \rightarrow x > \frac{-r}{1+a} \quad \textcircled{1}, \textcircled{2} \Rightarrow \frac{-r}{1+a} < x < \frac{r}{1-a}$$

$$\rightarrow \frac{r}{1-a} > \frac{-r}{1+a} \rightarrow \frac{r}{1-a} = r \Rightarrow 1-a=1 \rightarrow a=0 \rightarrow \boxed{a=1}$$

$$x > \frac{-r}{1+a} = \frac{-r}{1+1} = -\frac{r}{2} \rightarrow (b, r) = (-\frac{r}{2}, r) \Rightarrow a-b = 1 - (-\frac{r}{2}) = \boxed{\frac{r}{2}}$$

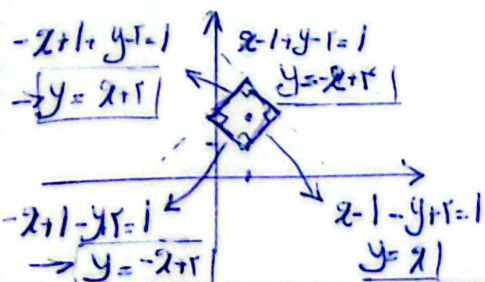
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$$|x-2| + |y-1| = 3$$



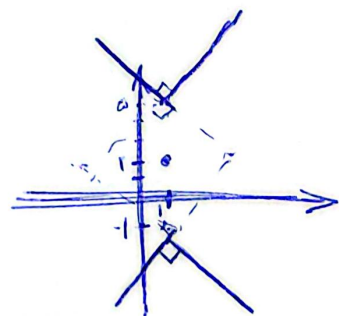
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$$|x-1| + |y-2| = 1$$



$$|x-1| - |y-2| = -3$$

$$\rightarrow |y-2| - |x-1| = 3$$



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$$y = rx + |ax+b|$$

$$R_f = IR$$

$$\text{if } ax+b \geq 0 \rightarrow rx + ax + b = (a+r)x + b$$

$$\text{if } ax+b < 0 \rightarrow rx - ax - b = (r-a)x - b$$

$$\rightarrow (a+r)/(r-a) < 0$$

$$\Rightarrow -a^2 + r^2 < 0 \rightarrow a^2 > r^2 \rightarrow \begin{cases} a > r \\ a < -r \end{cases}$$

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