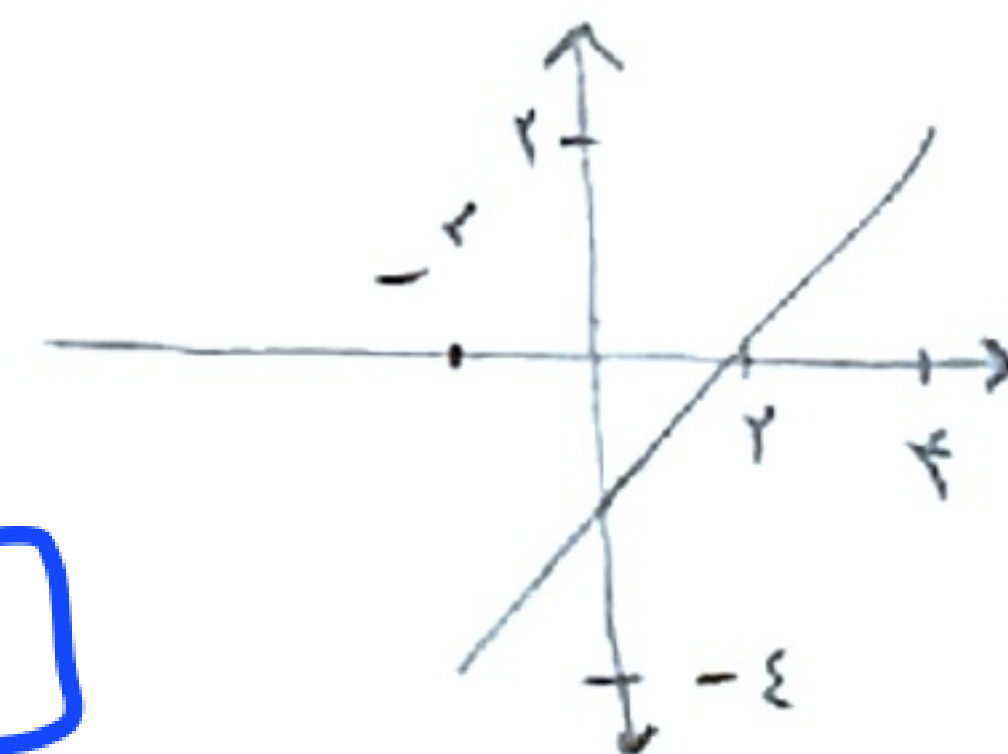


$f(x+1) + f(x+2) = 3x + f(3) \quad x=1 \quad f(3) = 3 + f(3) \rightarrow f(3) = 3$   
 $f(x) = ax + b \rightarrow 3a + b = 3 \quad x=0 \quad f(1) + f(2) = f(3)$   
 $a + b + 2a + b = 3 \rightarrow 3a + 2b = 3 \Rightarrow b = 0 \quad a=1$   
 $f(x) = x \quad f(n) = n$   
 $y = \sqrt{f(n) - n^2} \rightarrow \sqrt{3n - n^2} \rightarrow 3n - n^2 \rightarrow y_s = \frac{9}{4}$   
 $R_f = [0, \frac{9}{4}] \quad y = \sqrt{3n - n^2} \quad R_f = [0, \frac{1}{4}] \quad 3n - n^2 \leq \frac{9}{4}, 3n - n^2 \geq 0$

$f(n) = \frac{n(n^2 - 1)}{n+2} = \frac{n(n+1)(n-1)}{n+2} = \frac{n(n-1)}{n+2}, n \neq -2$   
 $x^2 - 2x \xrightarrow{x=-2} x^2 - 2x = +1$   
 $C = -1, a = b = -2$   
 $f(-1) = 3$   
 $f\left(\frac{a+b}{rc}\right) = f\left(\frac{-2-2}{2 \times (-1)}\right) = f(2) = (2)^2 - 2(2) = 0$   
 $2 \leq m = 1 \rightarrow a = 1, b = -2, c = -1$   
 $y_s = -1 \rightarrow R_f = [-1, +\infty) - \{1\}$   
 $x_s = 1 \quad D_f = R - \{-2\}$

$y = \sqrt[3]{x^3 + ax^2 + bx - 1} \xrightarrow{\text{تجرب و خطای}$   
 $y = \sqrt[3]{(x-2)^3} = x-2$   
 $(x-2)^3 = x^3 - 2x^2 + 4x - 8 \rightarrow a = -2, b = 4$   
 $D_f = [-2, 4] \quad y = \sqrt[3]{x^3 - 4x^2 + 12x - 8} \quad f(-2) = -4$   
 $R_f = [-4, 2] \quad a = -4, b = 12 \quad D_f = [-4, 12] \quad f\left(\frac{4}{3}\right) = 2$   
 $R_f = [-1, 1]$

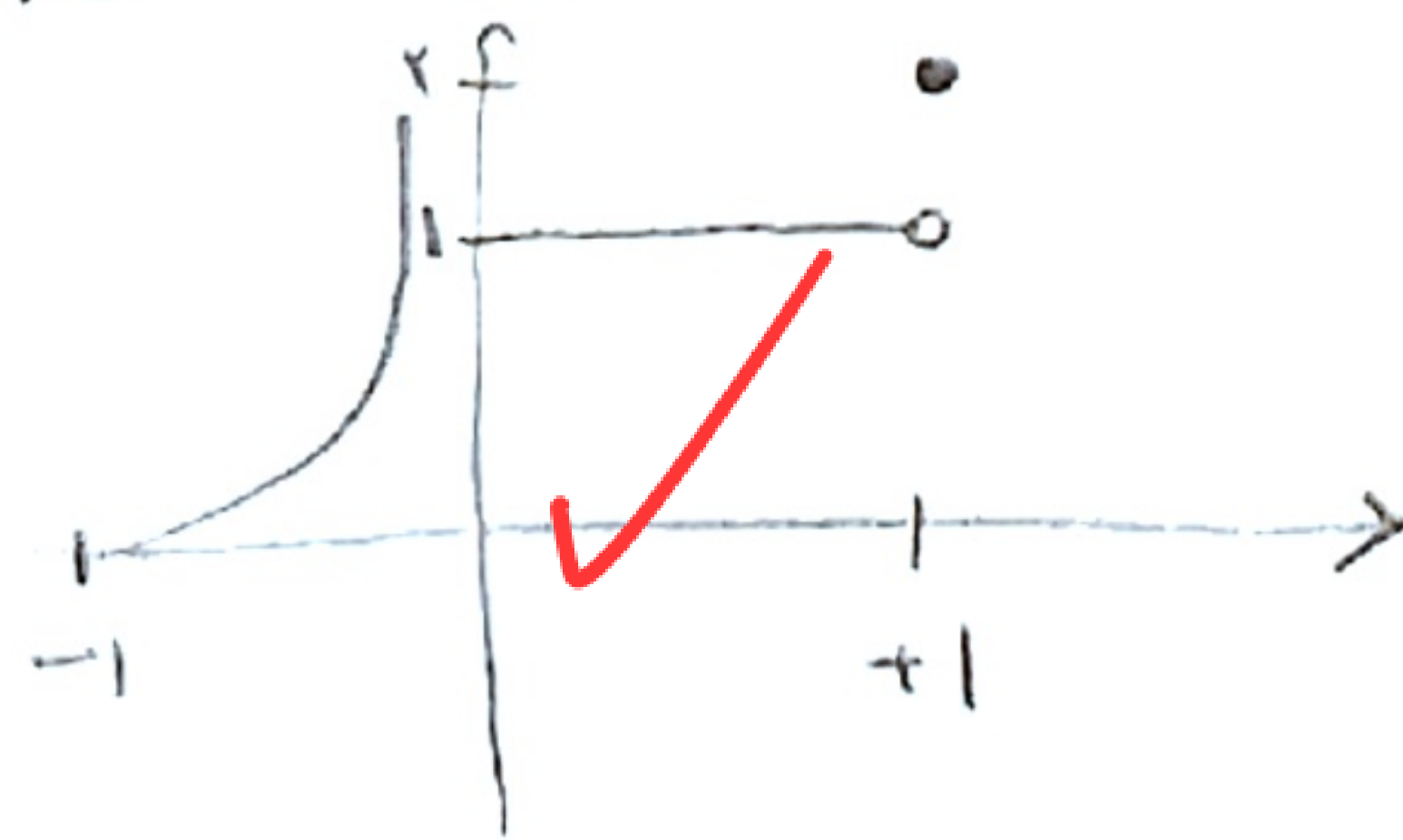


$D_{y_1} = [-1, +1) \quad R_{y_2} = [-1, +1) \quad a+b = 1+1 = 2$

$\frac{x+1}{1-x} \geq 0 \rightarrow \frac{-1}{-1} + \frac{+1}{0} \rightarrow D_{y_1} = [-1, +1)$

$\frac{ax^2 + b}{cx^2 + d} \xrightarrow{\text{مخرج و صورت به هم میزنیم}}$   
 $\frac{ax^2 + b}{cx^2 + d} \rightarrow x^{\max} = +\infty \rightarrow y = a$   
 $x^{\min} = 0 \rightarrow y = -\frac{1}{b}$   
 $R_{y_2} = [-\frac{1}{b}, a) \quad -\frac{1}{b} = -1 \rightarrow b = 1 \quad a = 1$

$f(x) = \frac{[x]}{x} + \frac{|x|}{x}$   
 $D_f = [-1, +1] - \{0\} \rightarrow -1 \leq x < 0 \rightarrow [x] = -1 \rightarrow -\frac{1}{x} - 1$   
 $0 < x < 1 \rightarrow [x] = 0 \rightarrow 0 + 1$   
 $x = 1 \rightarrow [x] = 1 \rightarrow \frac{1}{x} + 1$



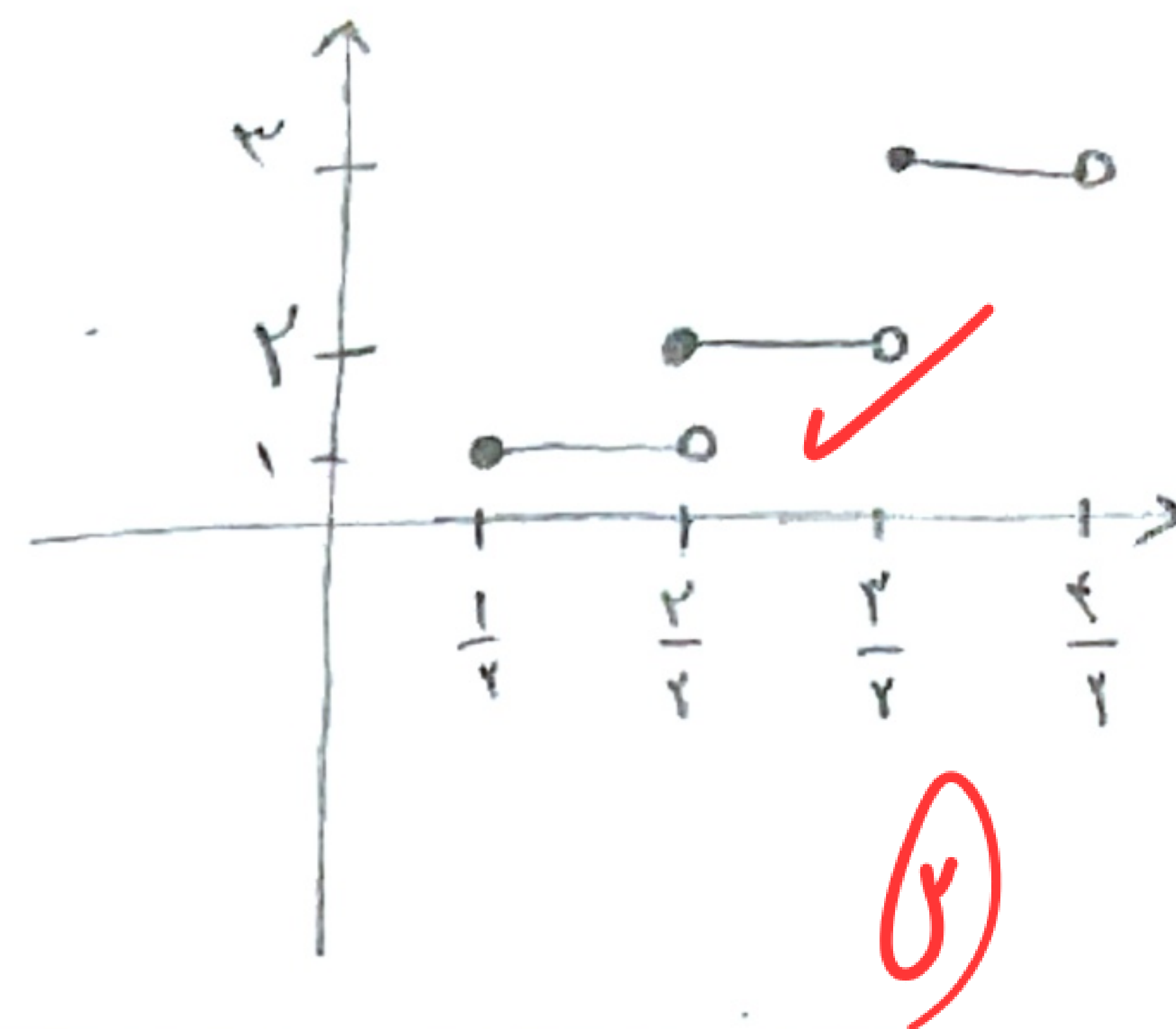


$$y = [x] + \left[x + \frac{1}{r}\right]$$

$$\frac{1}{r} \leq x < 1 \rightarrow [x] = 0, \left[x + \frac{1}{r}\right] = 1$$

$$1 \leq x < \frac{r}{2} \rightarrow [x] = 1, \left[x + \frac{1}{r}\right] = 1$$

$$\frac{r}{2} \leq x < r \rightarrow [x] = 1, \left[x + \frac{1}{r}\right] = 2$$



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$$\left| \frac{x+1}{ax+3} \right| < 1 \rightarrow |x+1| < |ax+3| \rightarrow (x+1)^2 < (ax+3)^2$$

$$(x+1)^2 - (ax+3)^2 < 0 \rightarrow (x+1+ax+3)(x+1-ax-3) < 0$$

$$((a+1)x+4)((1-a)x-2) < 0 \rightarrow \text{نقطه‌های بحرانی: } x = -\frac{4}{a+1}, x = \frac{2}{1-a}$$

$$a = -1 \rightarrow 4(2x-2) < 0 \rightarrow x-1 < 0 \rightarrow x < 1$$

$$a = 1 \rightarrow -2(2x+4) < 0 \rightarrow x+2 > 0 \rightarrow x > -2$$

اگر  $a=1$   $\frac{x+1}{x+3}$   
 $a=1 \rightarrow a-b=2$   
 $b=-1 \rightarrow a-b=2$

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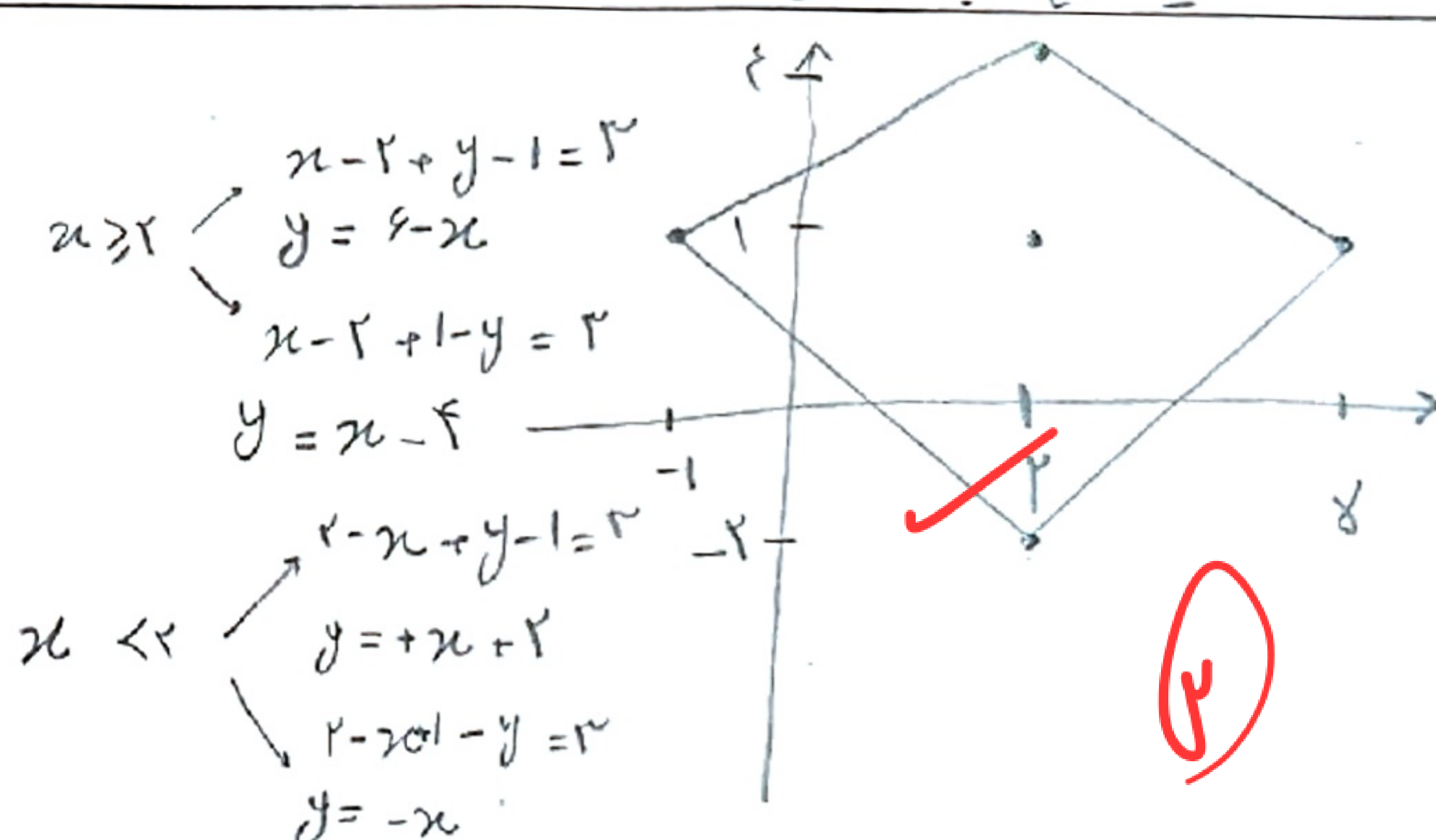
$$|x-2| + |y-1| = 3$$

$$x \geq 2 \rightarrow |x-2| = x-2$$

$$x < 2 \rightarrow |x-2| = 2-x$$

$$y \geq 1 \rightarrow |y-1| = y-1$$

$$y < 1 \rightarrow |y-1| = 1-y$$



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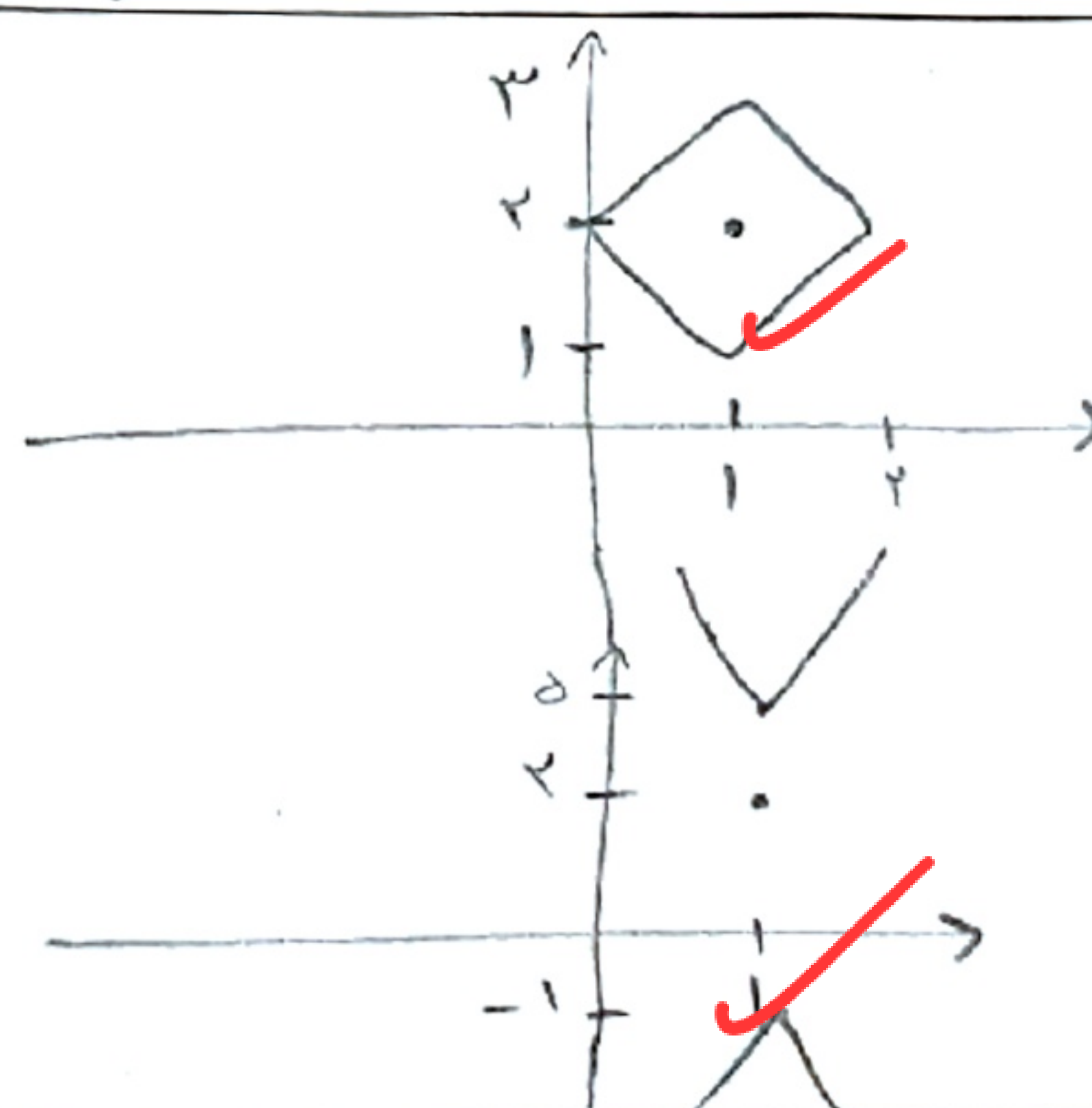
$$a) |x-1| + |y-2| = 1$$

$$\downarrow \quad \downarrow$$

$$x=1 \quad y=2$$

$$b) |x-1| - |y-2| = -3$$

$$= |y-2| - |x-1| = 3$$



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$$y = rx + |ax+b|$$

در دو صورت

$$y = \begin{cases} (r+a)x + b \\ (r-a)x - b \end{cases}$$

$$ax+b \geq 0 \rightarrow x \geq -\frac{b}{a}$$

$$ax+b < 0 \rightarrow x < -\frac{b}{a}$$

$$r+a > 0 \rightarrow a > -r$$

$$r-a > 0 \rightarrow r > a$$

$$r+a < 0 \rightarrow a < -r$$

$$r-a < 0 \rightarrow r < a$$

$$|a| < r, -r < a < r$$

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