

$$Dg = \{n \mid -n^2 + 4n - 4 > 0\}$$

$$\Rightarrow Dg = \{2\}$$

$$Dg \circ L = \{n \mid \sqrt{\log_2^{n-1}} = 2\} \Rightarrow n = 17$$

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$$L \circ g(n) = 2(n^2 - 4n + 4) - 2 = 2n^2 - 8n + 6$$

$$g \circ L(n) = (2n - 2)^2 - 2(2n - 2) + 1 = 4n^2 - 8n + 5$$

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

$$L \circ g(n) = g \circ L(n)$$

$$\Rightarrow 2n^2 - 8n + 6 = 4n^2 - 8n + 5 \Rightarrow 2n^2 - 2n - 1 = 0 \Rightarrow n = 1 \text{ or } n = -\frac{1}{2}$$

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$$g \circ L(n) = g(2n) = 2n^2 + 11$$

$$\Rightarrow g(t) = \frac{2}{5}t^2 + 11$$

$$g(n-1) = \frac{2}{5}(n-1)^2 + 11 \Rightarrow \text{کمترین مقدار تابع} = 11$$

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$$L(g(n)) = 3g(n) - 4 = 3n^2 - 6n - 1$$

$$\Rightarrow g(n) = (n-1)^2$$

$$g \circ L(n) = g(3n-4) = (3n-4-1)^2$$

$$g \circ L(n) = (3n-5)^2 = 9n^2 - 30n + 25$$

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$$L-g(n) = -n + 2$$

$$3g(n) = 3L(n) - 14$$

$$3L(n) - 3g(n) - 14 = 0$$

$$3[L(n) - g(n)] + L(n) - 14 = 0$$

$$3(-n+2) + L(n) - 14 = 0 \Rightarrow L(n) = 2n+8 \Rightarrow g(n) = 3n-1$$

$$L \circ g(n) = L(3n-1) = 9n+2$$

$$L \circ g(a) = 0 \Rightarrow 9a+2=0$$

$$\Rightarrow a = -\frac{2}{9}$$

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$f(n) = r^n$ $g(n) = \log_r n$ $f \circ g(n) = r^{r(\log_r n)} = n^r$ $g \circ f(n) = \log_r r^n = rn$ $f \circ g(n) \leq g \circ f(n)$ $\Rightarrow n^r \leq rn$ $\Rightarrow n^r - rn \leq 0$ $\Rightarrow 0 \leq n \leq r$	۶
$f\left(n - \frac{r}{n}\right) = n^r + \frac{r}{n^r} - r + n \cdot \frac{r}{n}$ $n \cdot \frac{r}{n} = t \quad \underbrace{\frac{r}{n^r}}_{t^r} \quad \underbrace{-r}_{t}$ $f(t) = t^r + t$	۷
$f \circ g(n) = \log_r^{1n - n^r}$ $A = D_{f \circ g} = \{n \mid 1n - n^r > 0\} = (0, 1)$ $B = R_{f \circ g} = (-\infty, \log_r^{1 - r^r}] = (-\infty, r]$ $A \cap B = [r, 1)$ دارای یک عضو صحیح	۸
$f \circ f(n) = 1 - r f^r(n)$ $f(n) = t$ $\Rightarrow f(t) = 1 - r t^r$ $h(g(n)) = f(n)$ $(n-1)^r + r(n-1) + 1 = 1 - r n^r$ $\Rightarrow n^r = -2n + r$ تنها یک ریشه	۹
$f(n) = n \cdot \frac{r}{n}$ $g \circ f(n) = a n^r + rn$ $g\left(n - \frac{r}{n}\right) = a n^r + rn$ $n = -1 \Rightarrow g(r) = -a - r = 1 \Rightarrow a = -10$ هیچ معادلی برای a ممکن نیست $n = r \Rightarrow g(r) = r^r a + 1 = 1 \Rightarrow a = 0$	۱۰