

$$D_{g \circ f} = \{x \in D_f \mid f(x) \in D_g\} \quad \left\{ D_{g \circ f} = \{1\} \right\} \leftarrow \boxed{x=1} \\ \rightarrow \log_2(x-1) \geq 0 \rightarrow x-1 \geq 1 \rightarrow x \geq 2 \\ x-1 > 0 \rightarrow x > 1 \quad \rightarrow D_f = [2, +\infty) \quad \log_2(x-1) = 2 \\ -2^x + 2x - 4 \geq 0 \rightarrow (x-2)' \leq 0 \rightarrow x=1 \rightarrow \boxed{D_{g \circ f} = \{2\}} \rightarrow \sqrt{\log_2(x-1)} = 2$$

$$f \circ g(x) = g \circ f(x) \rightarrow f(2^x - 2x + 1) = g(2x - 5) \\ \rightarrow 2(2^x - 2x + 1) - 5 = (2x - 5)^2 - 2(2x - 5) + 1 \\ \rightarrow 2 \cdot 2^x - 2 \cdot 2x + 2 - 5 = 4x^2 - 20x + 25 - 4x + 10 + 1 \\ 2 \cdot 2^x - 4x - 3 = 4x^2 - 24x + 36 \\ 2 \cdot 2^x - 4x - 3 = (2x-3)^2 - 2(2x-3) - 5 = 4x^2 - 12x + 9 - 4x + 6 - 5 = 4x^2 - 16x + 10$$

$$g(f(x)) = g(2x) = 2x^2 + 11 \quad \xrightarrow{\substack{2x=t \\ x=\frac{t}{2}}} g(t) = \frac{t^2}{4} + 11 \\ \rightarrow g(x) = \frac{x^2}{4} + 11 \\ g(x-1) = \frac{x^2}{4} + 11 \quad \xrightarrow{\text{معمولاً مقدار ۱۱}} g(1-1) = g(0) = 11 \\ \text{که حافظ به اشتباه می شود} \quad x=1$$

$$f(x) = 2x - 5 \\ f \circ g(x) = 2x^2 - 4x - 1 \\ f(g(x)) = 2x^2 - 4x - 1 \rightarrow 2t - 5 = 2x^2 - 4x - 1 \rightarrow 2t = 2x^2 - 4x + 4 \\ \rightarrow t = x^2 - 2x + 2 \quad \Rightarrow \boxed{g(x) = x^2 - 2x + 2}$$

$$2g(x) = 3 + f(x) - 14 \\ \rightarrow f(x) = \frac{2}{3}g(x) + \frac{11}{3} \\ \rightarrow f(x) = \frac{2}{3}(2x-1) + \frac{11}{3} \Rightarrow \boxed{f(x) = 2x + 3} \\ f \circ g = f(2x-1) = 2(2x-1) + 3 = 4x + 1 \rightarrow f \circ g(x) = 0 = 4x + 1 \rightarrow x = -\frac{1}{4} \Rightarrow \boxed{a = -\frac{1}{4}}$$

$$g(x) = \log_3 x \rightarrow x > 0$$

$$f(x) = 9^x = 3^{2x}$$

$$f \circ g(x) \neq g \circ f(x)$$

$$3^{2 \log_3 x} \neq \log_3 3^{2x}$$

$$\rightarrow 3^{(\log_3 x)^2} \neq x \rightarrow x^2 \neq x$$

$$\rightarrow x^2 - x \neq 0 \xrightarrow{+4-4} (x-2)(x+1) \xrightarrow{ND} (x-2)(x+1) \rightarrow x=1, 2 \in \mathbb{R}$$

$$f\left(\frac{x^2-1}{2}\right) = x^2 + x - 1 - \frac{x}{2} + \frac{1}{2^x} \rightarrow f\left(x - \frac{x}{2}\right) = \left(x^2 + \frac{x}{2} - 1\right) + \left(x - \frac{x}{2}\right)$$

$$\frac{x^2-1}{2} = x - \frac{x}{2} = t \rightarrow t^2 = x^2 + \frac{x}{2} - 1 \rightarrow f(t) = (t^2) + (t)$$

$$\rightarrow f(t) = t^2 + t \rightarrow \underline{f(x) = x^2 + x}$$

$$f \circ g = \log_3 12 - 2^x$$

$$12 - 2^x > 0 \rightarrow x \in (0, 1) \rightarrow D_{f \circ g} = (0, 1) = A$$

$$R_{f \circ g}: g(x) = -x^2 + 12 \xrightarrow{\max} x = \frac{-b}{2a} = 3 \rightarrow g(3) = -14 + 12 = -2$$

$$f(g(x)) = \log_3 g(x) \xrightarrow{g(x) > 0} B = (-\infty, 3] \rightarrow A \cap B = (0, 1) \cap (-\infty, 3] = (0, 1] \rightarrow \{1, 0.5, 0.25, \dots\}$$

$$f \circ f(x) = 1 - 3f^2(x)$$

$$g(x) = x - 1$$

$$h(x) = x^2 + 5x + 1$$

$$h(g(x)) = (x-1)^2 + 5(x-1) + 1 = x^2 - 2x + 1 + 5x - 5 + 1 = x^2 + 3x - 3 = f(x), \quad f(t) = t^2 - 3t + 5t - 2$$

$$\rightarrow f(x) = 0$$

$$f \circ f = f(f(x)) \xrightarrow{f(x)=t} f(t) = 1 - 3t^2$$

$$\boxed{2b \text{ و } 2c}$$

$$f(x) = 2 - \frac{x}{2}$$

$$g \circ f(x) = 9x^2 + 9x$$

$$g\left(\frac{1}{2}\right) = 1$$

$$2 - \frac{x}{2} = 2 \rightarrow x^2 - 2x - 2 = 0 \rightarrow x = -1, 4$$

$$x = -1 \rightarrow -a - 2 = 1 \rightarrow a = -3$$

$$x = 4 \rightarrow 4 + a + 1 = 1 \rightarrow a = -6$$