

$$D_f: x > 1 \text{ و } \sqrt{x-1} \geq 0 \Rightarrow x-1 \geq 1 \Rightarrow x \geq 2 \Rightarrow D_f = [2, +\infty)$$

$$D_g: -x^2 + 5x - 4 \geq 0 \Rightarrow x^2 - 5x + 4 \leq 0 \Rightarrow (x-2)(x-1) \leq 0 \Rightarrow x=2 = D_g$$

$$D_{g \circ f} = \{x \in D_f / f(x) \in D_g\}$$

$$\textcircled{1} x \geq 2$$

$$\sqrt{x-1} = 2 \Rightarrow \sqrt{x-1} = 2 \Rightarrow x-1 = 4 \Rightarrow x = 5$$

$$\textcircled{2} \Rightarrow D_{g \circ f} = \{1, 5\}$$

$$f(g(x)) = f(x^2 - 5x + 4) = 2(x^2 - 5x + 4) - 1 = 2x^2 - 10x + 7$$

$$g(f(x)) = g(2x^2 - 10x + 7) = (2x^2 - 10x + 7)^2 - 5(2x^2 - 10x + 7) + 4 = 4x^4 - 20x^3 + 29x^2 - 10x + 1$$

$$2x^2 - 10x + 7 = 4x^4 - 20x^3 + 29x^2 - 10x + 1 \Rightarrow 4x^4 - 22x^3 + 22x^2 - 1 = 0$$

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = (10)^2 - 2 \cdot 1 = 98$$

$$\begin{cases} \alpha + \beta = -\frac{b}{a} = 10 \\ \alpha\beta = \frac{c}{a} = -\frac{1}{4} \end{cases}$$

$$g(f(x)) = 4x^4 + 1 \quad f(x) = 2x \Rightarrow g(2x) = 4x^4 + 1$$

$$2x = t \Rightarrow \frac{t}{2} = x \Rightarrow g\left(\frac{t}{2}\right) = 4\left(\frac{t}{2}\right)^4 + 1 \Rightarrow g(x) = 4x^4 + 1$$

$$g(x-1) = 4(x-1)^4 + 1 \xrightarrow{\text{برای اینکه حداقل با } x=1 \text{ باشد}} 4 \times 0 + 1 = 1$$

$$f(x) = 3x - 4 \quad f \circ g = 3x^2 - 4x - 1 \quad f(g) = 3g - 4 = 3x^2 - 4x - 1$$

$$\Rightarrow 3g = 3x^2 - 4x + 1 \Rightarrow g = x^2 - \frac{4}{3}x + \frac{1}{3}$$

$$g(f(x)) = g(3x - 4) = (3x - 4)^2 - 4(3x - 4) + 1 = 9x^2 - 12x + 7$$

$$f(x) - g(x) = -x + 4 \xrightarrow{\times 2} 2f(x) - 2g(x) = -2x + 8 \quad \textcircled{1}$$

$$2f(x) - 2g(x) = 1 \quad \textcircled{2}$$

$$-f(x) = -2x - 4 \Rightarrow f(x) = 2x + 4$$

$$f(g(x)) = f(3x - 1) = 2(3x - 1) + 4 = 6x + 2$$

$$6x + 2 = 0 \Rightarrow x = -\frac{1}{3} = \alpha$$

برای بدست آوردن α قرار می دهیم $y=0$

$$f(x) = q^x g(x) = \log_y^x \quad f(g(x)) = f(\log_y^x) = q^{\log_y^x} = x^y$$

$$g(f(x)) = g(q^x) = \log_y^{q^x} = yx$$

$$f \circ g(x) \leq g \circ f(x) \Rightarrow x^y \leq yx \rightarrow x^y - yx \leq 0 \rightarrow x(x-y) \leq 0$$

$$\frac{0}{+} \frac{+}{-} \frac{-}{+} \rightarrow x \in [0, y] \rightarrow 0, 9, 1, 9, y \checkmark \rightarrow \text{مستقيم (م)} \quad \textcircled{y}$$

$$f\left(\frac{x^y - y}{x}\right) = x^y + x - \varepsilon - \frac{y}{x} + \frac{\varepsilon}{x^y} = x^y + \frac{\varepsilon}{x^y} - \varepsilon + x - \frac{y}{x}$$

$$= \frac{x^y - \varepsilon x^y + \varepsilon}{x^y} + \frac{x^y - y}{x} \Rightarrow f(x) = x^y + x \quad \textcircled{y}$$

$$f(x) = \log_y^x \quad g(x) = 1/x - x^y \rightarrow f(g(x)) = \log_y^{1/x - x^y} \quad \textcircled{1}$$

$$D_{f \circ g}: 1/x - x^y > 0 \rightarrow x(-x^y + 1) > 0 \rightarrow \frac{0}{-} \frac{+}{+} \frac{-}{-} \rightarrow D_{f \circ g} = (0, 1) = A$$

$$\begin{array}{c} \text{to } g(x) \\ \text{ } \end{array} \Rightarrow R_{f \circ g} = |R = B \Rightarrow A \cap B = (0, 1) \rightarrow \text{مستقيم (م)} \quad \textcircled{y}$$

$$A \cap B = (0, +\infty) \rightarrow \text{مستقيم (م)} \quad \textcircled{y}$$

$$f \circ f(x) = 1 - x^y f'(x) \quad g(x) = x - 1 \quad h(x) = x^y + yx + 1$$

$$f(f(x)) = 1 - x^y f'(x) \rightarrow f(x) = 1 - x^y \quad g(h(x)) = (x-1)^y + y(x-1) + 1$$

$$= x^y - yx^y + yx - 1 + yx - y + 1 = x^y - yx^y + 2yx - y$$

$$h(g(x)) = f(x) \Rightarrow x^y - yx^y + 2yx - y = 1 - x^y \Rightarrow x^y + 2yx - y = 0 \quad \textcircled{y}$$

$$\text{مستقيم} \quad yx^y + 2y > 0 \Rightarrow \text{مستقيم (م)} \quad \textcircled{y}$$

$$f(x) = x - \frac{\varepsilon}{x} \quad g \circ f(x) = ax^y + yx \rightarrow g\left(x - \frac{\varepsilon}{x}\right) = ax^y + yx$$

$$g(y) = 1 \rightarrow x - \frac{\varepsilon}{x} = y \Rightarrow x^y - yx - \varepsilon = 0 \rightarrow (x - \varepsilon)(x + 1) = 0 \quad \begin{cases} x = \varepsilon \\ x = -1 \end{cases}$$

$$x = \varepsilon \rightarrow 4\varepsilon a + 1 = 1 \rightarrow a = 0$$

$$x = -1 \rightarrow -a - y = 1 \rightarrow -a = 1 \rightarrow a = -1 \quad \textcircled{y}$$