۱- مبین شیبانی

۱۷٫۷۵

$$g\circ f(x) \{ x \in D_f , f(x) \in D_g \}$$

$$g(x) = \sqrt{-x^2 + 4x - 4}$$

$$-x^2 + 4x - 4 \geq 0 \qquad (x-2)^2 \leq 0 \to (x-2)^2 = 0$$

$$-(x-2)^2 \geq 0 \qquad x = 2$$

$$D_g = \{2\}$$

$$f(x) = \sqrt{\log_2(x-1)} \qquad \log_2(x-1) \geq 0 \qquad x-1 \geq 1 \qquad x \geq 2$$

$$x - 1 > 0 \qquad x > 1 \cap x \geq 2 \to x \geq 2$$

$$\sqrt{\log_2(x-1)} = 2 \qquad \log_2(x-1) = 4 \qquad x = 17$$

$$D_{g\circ f} = \{17\}$$

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$$f(x) = 2x - 5$$

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$$g(x) = x^2 - 3x + 8$$

$$f\circ g(x) = g\circ f(x) \qquad 2g(x) - 5 = f(x)^2 - 3f(x) + 8$$

$$2(x^2 - 3x + 8) - 5$$

$$2x^2 - 6x + 16 - 5 = (2x-5)^2 - 3(2x-5) + 8$$

$$\cancel{2x^2} - \cancel{6x} + 9 = 4x^2 - 20x + 25 - \cancel{6x} + 15 + 8$$

$$2x^2 - 20x + 39 = 0 \rightarrow 2x^2 - 20x + 39 = 0$$

$$400 - 4(2)(39) = 4(100 - 78) = 88 \qquad \frac{20 \pm \sqrt{88}}{4} =$$

$$\left(\frac{20+\sqrt{88}}{4}\right)^2 + \left(\frac{20-\sqrt{88}}{4}\right)^2 = \frac{400 + 40\sqrt{88} + 88}{16} + \frac{400 - 40\sqrt{88} + 88}{16} = \frac{2(488)}{\cancel{16}_8}$$

$$\alpha^2 + \beta^2 = S^2 - 2P = 100 - 37 = \boxed{63}$$

$$\boxed{\frac{488}{8} = 61}$$

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$g \circ f(x) = 5x^2 + 11 \rightarrow g(f(x)) \rightarrow g(2x) = 5x^2 + 11$

$f(x) = 2x$

$2x \rightarrow t$ $\quad 4x^2 + x^2 + 11$

$(2x)^2 + \left(\frac{2x}{2}\right)^2 + 11 \rightarrow t^2 + \left(\frac{t}{2}\right)^2 + 11$

$g(t) = \frac{5t^2}{4} + 11$

$g(x) = \frac{5x^2}{4} + 11$ دامنه همواره مثبت

$g(x-7) = \frac{5(x-7)^2}{4} + 11$

$D_g = \mathbb{R}$

$R_g = [11, +\infty)$

$\boxed{\min g(x) = 11}$

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$f(x) = 3x - 4$

$f \circ g(x) = 3x^2 - 6x - 1 \rightarrow f(g(x)) = 3x^2 - 6x - 1 \rightarrow 3g - 4 = 3x^2 - 6x - 1$

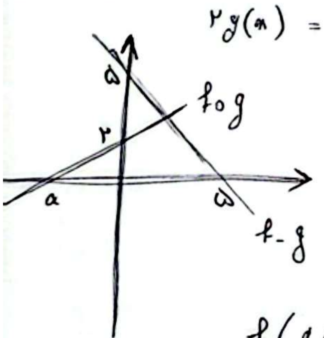
$3g = 3x^2 - 6x + 3$

$g(x) = x^2 - 2x + 1$

$g \circ f(x) = (3x-4)^2 - 2(3x-4) + 1$

$\boxed{g \circ f(x) = 9x^2 - 30x + 25}$

2



$2g(x) = 3f(x) - 14$

$f(0) - (-1) = 5$

$f(0) - g(0) = 5$

$3f(0) = 3g(0) + 15$

$3g(0) + 15 = 2g(0) + 14$ $\quad g(0) = -1$

$f(5) - g(5) = 0$

$3f(5) = 3g(5)$

$2g(5) + 14 = 3g(5)$

$g(5) = 14 \quad f(5) = 14$

$f(g(0)) = 2 \rightarrow f(-1) = 2$

$\begin{vmatrix} -1 \\ 2 \end{vmatrix}$ $\begin{vmatrix} 0 \\ 4 \end{vmatrix}$ $\begin{vmatrix} 5 \\ 14 \end{vmatrix}$ $f(x) = 2x + 4$

$g(a) = -2$ $\quad x = -\frac{1}{3}$

$3x - 1 = -2$

$\begin{vmatrix} 0 \\ -1 \end{vmatrix}$ $\begin{vmatrix} 5 \\ 14 \end{vmatrix}$ $g(x) = 3x - 1$

$2x + 4 = 0$

$2x = -4$

$x = -2$

$\boxed{a = -\frac{1}{3}}$

2

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$f(x) = 9^x$

$g(x) = \log_3 x$

$f \circ g(x) \leq g \circ f(x)$

$f(g) \leq g(f)$ $\quad f(\log_3 x) \leq g(9^x)$

$9^{\log_3 x} \leq \log_3 9^x$

$x^{\log_3 9} \leq x \log_3 9$

$x^2 \leq 2x$

$x^2 - 2x \leq 0 \quad x(x-2) \leq 0$

	0		2	
+	0	−	0	+

$0 \leq x \leq 2, \; x \in \mathbb{Z} = \{0, 1, 2\}$

2

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$f\left(\frac{x^2-2}{x}\right) = x^2 + x - 4 + \frac{2}{x} + \frac{4}{x^2}$ ۱

$f(x) = x^2 + x$

$f\left(x - \frac{2}{x}\right) = x^2 + \frac{4}{x^2} - 4 + x + \frac{2}{x} = \left(x - \frac{2}{x}\right)^2 + \sqrt{\left(x-\frac{2}{x}\right)^2 + 8}$

$\left(x + \frac{2}{x}\right)^2 = x^2 + 4 + \frac{4}{x^2} = \left(x - \frac{2}{x}\right)^2 + 8 \qquad \boxed{f(x) = t^2 + \sqrt{t^2+8}}$

$\left(x + \frac{2}{x}\right)^2 = \left(x - \frac{2}{x}\right)^2 + 8 \qquad f\left(x - \frac{2}{x}\right) = x^2 + \frac{4}{x^2} - 4 + x - \frac{2}{x}$

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$f(x) = \log_2 x$

$g(x) = 8x - x^2$

$f\circ g(x) = f(g(x)) \Rightarrow f(8x - x^2) = \log_2 (8x - x^2)$

$A = D_{f\circ g} = (0, 8)$

$B = R_{f\circ g} = (-\infty, 4]$

$f\circ g(x) = \log_2 (8x - x^2)$

$8x - x^2 > 0$

$x(8 - x) > 0 \qquad$ $0 \quad 8$: $- \quad + \quad -$

$D_{f\circ g} = (0, 8)$

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$\log_2 g(x) \to g(x) \to 0^+ = -\infty$

$\log_2 (g(x)) \to x_{\text{رأس}} = \frac{-b}{2a} = \frac{-8}{-2} = 4 \qquad \log_2 16 = 4$

$\boxed{(0,8) \cap (-\infty, 4] = (0, 4]}$ عضو صحیح ۴

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$f\circ f(x) = 1 - 3f^2(x) \qquad f(f(x)) = 1 - 3f^2(x) \qquad f(x) = 1 - 3x^2$

$g(x) = x - 1$

$h(x) = x^3 + 2x + 1$

$x^3 + 5x - 3 = 0 \qquad 3x^2 + 5$ ، ~~مثبت~~ صعودی

بنابراین تابع اصلی ~~...~~ صعودی است

$h(g(x)) = f(x) \Rightarrow (x-1)^3 + 2(x-1) + 1 = 1 - 3x^2$

$x^3 - \cancel{3x^2} + 3x - 1 + 2x - 2 + \cancel{1} = \cancel{1} - \cancel{3x^2}$

$x^3 + 5x - 3 = 0$

دارای ۱ ریشه است ۲

۱ ریشه حقیقی

۱۰-

$f(x) = x - \frac{۴}{x}$

$g(f) = ax^۳ + ۲x$ $\quad$ $g\left(x - \frac{۴}{x}\right) = ax^۳ + ۲x$

$x - \frac{۴}{x} = ۳$ $\quad$ $\frac{x^۲ - ۴}{x} = ۳$

$x^۲ - ۳x - ۴ = ۰$

$(x-۴)(x+۱) = ۰$

$x = ۴$

$x = -۱$

$ax^۳ + ۲x = ۸$

$-a - ۲ = ۸$

$-a = ۱۰$

$a = -۱۰$

$۶۴a + ۸ = ۸$

$۶۴a = ۰ \quad a = ۰$

$\boxed{a = -۱۰ \text{ ق ق}}$

$a = ۰$ ~~ق ق~~

ق ق

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