۱- مبین شهبازی

$$g\circ f(x)\{x\in D_f , f(x)\in D_g\}$$

$$g(x)=\sqrt{-x^2+4x-4}$$

$$-x^2+4x-4\geq 0 \qquad (x-2)^2\leq 0 \to (x-2)^2=0$$

$$-(x-2)^2\geq 0 \qquad x=2$$

$$D_g=\{2\}$$

$$f(x)=\sqrt{\log_2(x-1)} \qquad \log_2(x-1)\geq 0 \qquad x-1\geq 1 \qquad x\geq 2$$

$$x-1>0 \qquad x>1 \cap x\geq 2 \to x\geq 2$$

$$\sqrt{\log_2(x-1)}=2 \qquad \log_2(x-1)=4 \qquad x=17$$

$$D_{g\circ f}=\{17\}$$

۲-

$$f(x)=2x-5$$

$$g(x)=x^2-3x+8$$

$$f\circ g(x)=g\circ f(x) \qquad 2g(x)-5=f(x)^2-3f(x)+8$$

$$2(x^2-3x+8)-5$$

$$2x^2-6x+16-5=(2x-5)^2-3(2x-5)+8$$

$$\cancel{2x^2}-\cancel{6x}+9=4x^2-20x+25-\cancel{6x}+15+8$$

$$2x^2-20x+39=0$$

$$400-4(2)(39)=4(100-78)=88 \qquad \frac{20\pm\sqrt{88}}{4}=$$

$$\left(\frac{20+\sqrt{88}}{4}\right)^2+\left(\frac{20-\sqrt{88}}{4}\right)^2=\frac{400+40\sqrt{88}+88}{16}+\frac{400-40\sqrt{88}+88}{16}=\frac{2(488)}{\cancel{16}_{8}}$$

$$\boxed{\frac{488}{8}=61}$$

3-

$g\circ f(x) = 5x^2+11 \rightarrow g(f(x)) \rightarrow g(2x) = 5x^2+11$

$f(x) = 2x$

$g(x-7) = ?$

$2x \rightarrow t \qquad 4x^2 + x^2 + 11 \qquad (2x)^2 + \left(\frac{2x}{2}\right)^2 + 11 \rightarrow t^2 + \left(\frac{t}{2}\right)^2 + 11$

$g(x) = \frac{5x^2}{4} + 11$ دائما همواره مثبت

$g(t) = \frac{5t^2}{4} + 11$

$g(x-7) = \frac{5(x-7)^2}{4} + 11 \qquad D_g = \mathbb{R} \qquad R_g = [11, +\infty) \qquad \boxed{\min g(x) = 11}$

4

$f(x) = 3x-4$

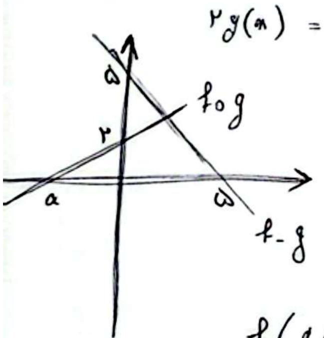
$f\circ g(x) = 3x^2-6x-1 \rightarrow f(g(x)) = 3x^2-6x-1 \rightarrow 3g-4 = 3x^2-6x-1$

$3g = 3x^2-6x+3$

$g(x) = x^2-2x+1$

$g\circ f(x) = (3x-4)^2 - 2(3x-4) + 1$

$\boxed{g\circ f(x) = 9x^2-30x+25}$



$3g(x) = 3f(x) - 14$

$f(0) - (-1) = 5$

$f(0) - g(0) = 5 \qquad 3f(0) = 3g(0) + 15 \qquad g(a) = -2 \quad a = -\frac{1}{3}$

$3x - 1 = -2$

$3g(0) + 15 = 3g(0) + 14 \qquad g(0) = -1 \qquad \begin{vmatrix} 0 \\ -1 \end{vmatrix} \begin{vmatrix} 5 \\ 14 \end{vmatrix} \qquad g(x) = 3x - 1$

$f(5) - g(5) = 0 \qquad 3f(5) = 3g(5) \qquad 3x + 4 = 0$

$3g(5) + 14 = 3g(5) \qquad g(5) = 14 \quad f(5) = 14 \qquad 3x = -4$

$x = - \qquad \boxed{a = -\frac{1}{3}}$

$f(g(0)) = 2 \rightarrow f(-1) = 2 \qquad \begin{vmatrix} -1 \\ 2 \end{vmatrix} \begin{vmatrix} 0 \\ 4 \end{vmatrix} \begin{vmatrix} 5 \\ 14 \end{vmatrix} \qquad f(x) = 2x + 4$

6-

$f(x) = 9^x$

$g(x) = \log_3 x$

$f\circ g(x) \leq g\circ f(x)$

$f(g) \leq g(f) \qquad f(\log_3 x) \leq g(9^x)$

$9^{\log_3 x} \leq \log_3 9^x \qquad x^2 \leq 2x$

$x^{\log_3 9} \leq x \log_3 9$

$x^2 - 2x \leq 0 \qquad x(x-2) \leq 0$

	0		2	
+	0	−	0	+

$0 \leq x \leq 2, \; x \in \mathbb{Z} = \{0, 1, 2\}$

-۷

$$f\left(\frac{x^2-2}{x}\right) = x^2 + x - 4 + \frac{2}{x} + \frac{4}{x^2}$$

$$\downarrow$$

$$f\left(x - \frac{2}{x}\right) = x^2 + \frac{4}{x^2} - 4 + x + \frac{2}{x} = \left(x - \frac{2}{x}\right)^2 + \sqrt{\left(x - \frac{2}{x}\right)^2 + 8}$$

$$\left(x + \frac{2}{x}\right)^2 = x^2 + 4 + \frac{4}{x^2} = \left(x - \frac{2}{x}\right)^2 + 8 \qquad \boxed{f(x) = t^2 + \sqrt{t^2+8}}$$

$$\left(x + \frac{2}{x}\right)^2 = \left(x - \frac{2}{x}\right)^2 + 8$$

-۸

$$f(x) = \log_2 x \qquad f\circ g(x) = f(g(x)) \Rightarrow f(8x - x^2) = \log_2^{8x-x^2}$$

$$g(x) = 8x - x^2$$

$$f\circ g(x) = \log_2^{8x-x^2}$$

$$A = D_{f\circ g} = (0,8)$$

$$8x - x^2 > 0$$

$$B = R_{f\circ g} = (-\infty, 4]$$

$$x(8-x) > 0$$

x		0		8	
	$-$		$+$		$-$

$$D_{f\circ g} = (0,8)$$

$$\cancel{\log_2^{8x-x^2}} \qquad \cancel{8x \ldots 4}$$

$$\log_2 g(x) \to g(x) \to 0^+ = -\infty$$

$$\log_2(g(x)) \to x_{\text{رأس}} = \frac{-b}{2a} = \frac{-8}{-2} = 4 \qquad \log_2 16 = 4$$

$$\boxed{(0,8) \cap (-\infty, 4] = (0,4]}$$

-۹

$$f\circ f(x) = 1 - 3f^2(x) \qquad f(f(x)) = 1 - 3f^2(x) \qquad f(x) = 1 - 3x^2$$

$$g(x) = x - 1$$

$$h(x) = x^3 + 2x + 1$$

$$x^3 + 5x - 3 = 0 \qquad 3x^2 + 5$$

~~...~~ صعودی

بنابراین تابع اصلی نیز ~~...~~ صعودی است

دارای ۳ ریشه است

۱ ریشه حقیقی

$$h(g(x)) = f(x) \Rightarrow (x-1)^3 + 2(x-1) + 1 = 1 - 3x^2$$

$$x^3 - \cancel{3x^2} + 3x - 1 + 2x - 2 + \cancel{1} = \cancel{1} - \cancel{3x^2}$$

$$x^3 + 5x - 3 = 0$$

-۱.

$$f(x) = x - \frac{4}{x}$$

$$g(f) = ax^3 + 2x \qquad g\left(x - \frac{4}{x}\right) = ax^3 + 2x$$

$$x - \frac{4}{x} = 3 \qquad \frac{x^2 - 4}{x} = 3$$

$$x^2 - 3x - 4 = 0$$

$$(x-4)(x+1) = 0$$

$$x = 4$$

$$x = -1$$

$$ax^3 + 2x = 8$$

$$-a - 2 = 8$$

$$-a = 10$$

$$a = -10$$

$$64a + 8 = 8$$

$$64a = 0 \qquad a = 0$$

$\boxed{a = -10 \text{ اقق}}$

$a = 0$ عقق