نام و نام خانوادگی .... پاسخنامه تشریحی تکلیف شماره ..۸.... کلاس ...یازدهم A...

۱

$g(x) = \sqrt{-(x-2)^2} \Rightarrow D_g = \{2\} \Rightarrow f(x) = \sqrt{\log_2^{(x-1)}} = 2$

$\Rightarrow \log_2^{x-1} = 4 \Rightarrow x = 17 \Rightarrow D_{gof} = \{17\}$

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$fog = 2x^2 - 6x + 14$, $gof = 4x^2 - 26x + 51$

$\Rightarrow 2x^2 - 6x + 11 = 4x^2 - 26x + 51 \Rightarrow 2x^2 - 20x + 47 = 0$

$\alpha + \beta = 10 \quad \alpha\beta = \frac{47}{2} \Rightarrow (\alpha+\beta)^2 = \alpha^2 + \beta^2 + \underbrace{2\alpha\beta}_{47} = 100$

$\Rightarrow \boxed{\alpha^2 + \beta^2 = 53}$

۳

$g(2x) = 5x^2 + 11 \Rightarrow 2x = t \Rightarrow x = \frac{t}{2} \Rightarrow g(t) = 5\frac{t^2}{4} + 11$

$\Rightarrow g(x) = 5\frac{x^2}{4} + 11 \Rightarrow g(x-7) = 5\frac{(x-7)^2}{4} + 11$

$= 5\frac{x^2 - 14x + 49}{4} + 44 = \frac{5}{4}x^2 - \frac{70}{4}x + \frac{5\times 93}{4}$

$x_{min} = 7 \Rightarrow \boxed{y_{min} = 11}$

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$3g(x) - 4 = 3x^2 - 6x - 1 \Rightarrow 3g(x) = 3x^2 - 6x + 3$

$\Rightarrow g(x) = x^2 - 2x + 1 \Rightarrow g(3x-4) = 3(3x-4)^2 - 6(3x-4) + 3$

$= \boxed{27x^2 - 90x + 75 = gof(x)}$

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$g(x) = \frac{3}{2}f(x) - 7$, $f(x) - \frac{3}{2}f(x) + 7 = -x + 5$

$\Rightarrow -\frac{1}{2}f(x) + 7 = -x + 5 \Rightarrow f(x) = 2x + 4$

$fog(x) = \frac{-2x}{a} + 2 \Rightarrow 2(gx) + 4 = \frac{-2x}{a} + 2 \Rightarrow g(x) = -\frac{x}{a} - 1$

$g(x) = \frac{3}{2} \times (2x + 4) - 7 = 3x - 1 \Rightarrow 3x - 1 = -\frac{x}{a} - 1$

$\Rightarrow \boxed{a = -\frac{1}{3}}$

۶

$f\circ g(x) = 9^{\log_3 x} = x^{\log_3 9} = x^2$, $g\circ f(x) = \log_3 9^x = x\log_3 9 = 2x$

$x^2 \le 2x \Rightarrow x^2 - 2x \le 0 \Rightarrow x(x-2) \le 0$

	0		2	
+		−		+

$\Rightarrow 0 \le x \le 2$, $x \ne 0 \Rightarrow x = (0,2]$

۷

$f(x - \frac{2}{x}) = x^2 + \frac{4}{x^2} - 4 + x - \frac{2}{x}$

$= f(x - \frac{2}{x}) = (x - \frac{2}{x})^2 + x - \frac{2}{x}$, $x - \frac{2}{x} = t$

$\Rightarrow f(t) = t^2 + t \Rightarrow f(x) = x^2 + x$

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$f\circ g(x) = \log_2^{8x - x^2} \Rightarrow D_{f\circ g} = 8x - x^2 > 0$

$\Rightarrow x(8-x) > 0 \Rightarrow$

	0		8	
−		+		−

$\Rightarrow D_{f\circ g} = (0,8) = A$

$x_{max} = 4 \Rightarrow y_{max} = \log_2^{16} = 4$

$\Rightarrow R_{f\circ g} = (-\infty, 4] = B \Rightarrow B \cap A = (0,4]$

۹

$f(f(x)) = 1 - 3f^2(x) \Rightarrow f(x) = t \Rightarrow f(t) = 1 - 3t^2 \Rightarrow$

$f(x) = 1 - 3x^2$, $h\circ g(x) = x^3 - 3x^2 + 5x$

$\Rightarrow x^3 - 3x^2 + 5x = 1 - 3x^2 \Rightarrow x^3 + 5x = 1$

دو تابع اکیداً صعودی جمع شد. و چون بردش R است، فقط یک ریشه دارد.

۱۰

$g(x - \frac{4}{x}) = ax^3 + 2x \Rightarrow x - \frac{4}{x} = 3 \Rightarrow \frac{x^2 - 4}{x} = 3$

$\Rightarrow x^2 - 3x - 4 = 0 \Rightarrow x = -1 , x = 4$

حالت اول: $x = -1 \Rightarrow g(3) = -a - 2 = 8 \Rightarrow a = -10$

حالت دوم: $x = 4 \Rightarrow g(3) = 64a + 8 = 8 \Rightarrow a = 0$