علیرضا آقاجانی

19,25

(1)

$\{x \in D_f \mid f(x) \in D_g\}$ الف $\rightarrow D_f \rightarrow \sqrt{\log_2^{x-1}} \rightarrow x-1>0 \rightarrow \boxed{x>1}$ (I)

$\log_2^{x-1} \geq 0 \rightarrow x-1 \geq 2^0 \rightarrow \boxed{x \geq 2}$ (II)

$I \cap II \rightarrow \boxed{x \geq 2} = D_f$

ب ↓ $D_g \rightarrow \sqrt{-(x-2)^2} \rightarrow x=2$ تنها حالتی که درست ↓ $\ominus(x-2)^2 \geq 0 \rightarrow \boxed{x=2}$

$\sqrt{\log_2^{x-1}} = 2 \rightarrow \log_2^{x-1} = 4 \rightarrow x-1 = 2^4 = 16 \rightarrow \boxed{x=17}$

$\rightarrow \{17\} \cap [2,+\infty) \rightarrow \{17\} \rightarrow \boxed{D_{gof}}$ ✓

2

(2)

$fog(x) \rightarrow f(x^2-3x+8) \rightarrow 2(x^2-3x+8)-5 \rightarrow 2x^2-6x+11$

$gof(x) \rightarrow g(2x-5) \rightarrow (2x-5)^2 - 3(2x-5)+8 \rightarrow 4x^2-26x+48$

$4x^2-20x+25-6x+15+8$

$\rightarrow 2x^2-6x+11 = 4x^2-26x+48 \rightarrow 2x^2-20x+37 \rightarrow \boxed{\alpha^2+\beta^2 \rightarrow S^2-2P}$

$10 \leftarrow S = -\frac{b}{a}$

$18{,}5 \leftarrow P = \frac{c}{a}$

$100 - \frac{2(18{,}5)}{37} = 63 \rightarrow$ جواب ✓

2

(3)

$gof(x) = 5x^2+11 \rightarrow g(\underbrace{2x}_{t}) = 5x^2+11 \rightarrow 2x=t \rightarrow x=\frac{t}{2} \rightarrow g(t) = 5(\frac{t}{2})^2+11$

$\rightarrow g(t) = \frac{5}{4}t^2+11 \xrightarrow[\text{ورودی}]{(t-7)} g(t-7) = \frac{5}{4}(t-7)^2+11 \rightarrow \boxed{\frac{5}{4}t^2 - \frac{35}{2}t + 61{,}25+11}$ ✓

$t^2-14t+49$

$y_s \xrightarrow{x=7} \frac{5}{4}(49) - \frac{35}{2}(7) + 72{,}25 \leftarrow \textcircled{7} = \frac{\frac{35}{2}}{\frac{5}{2}} \leftarrow -\frac{b}{2a} \leftarrow x_s$

$61{,}25 - 129{,}5 + 72{,}25 = \textcircled{4}$ جواب ✓

$R_{min} = 11$

1,5

(4)

$fog \rightarrow 3x^2-6x-1 \longrightarrow 3g(x) - 4 = 3x^2-6x-1 \rightarrow 3g(x) = \frac{3x^2-6x+3}{3} \rightarrow \boxed{g(x) = x^2-2x+1}$ (I)

$gof(x) \rightarrow g(3x-4) \xrightarrow{I} (3x-4)^2 - 2(3x-4)+1 \rightarrow \boxed{9x^2-30x+25}$ ✓

$9x^2-24x+16-6x+8+1$

2

(5)

$f-g(x) \rightarrow$ شیب $\rightarrow -\frac{5}{5} \rightarrow \textcircled{-1} \xrightarrow{\text{فاصله}} \boxed{-x+5}$

$2g(x) = 3f(x) - 14 \rightarrow -f(x)+14 = 2f(x) - 2g(x)$

$2(f-g)(x)$ نتیجه

$\Rightarrow -f(x)+14 = -2x+10$

$\rightarrow 2x+4 = f(x)$

$2x+4 - g(x) = -x+5 \rightarrow -g(x) = -3x+1 \xrightarrow{-x+5} \boxed{g(x) = 3x-1}$

موجود در تاخیر ۱۲ است

$fog(x) \rightarrow 2(3x-1)+4 \rightarrow 6x+2 \longrightarrow \boxed{\text{شیب} = 6} \rightarrow \frac{2}{\Delta x} = 6 \rightarrow \Delta x = \frac{1}{3} \xrightarrow{\text{نتیجه}} a = \ominus\frac{1}{3}$ ✓

2

(۶)

$gof(x) \to g(9^x) \to \boxed{\log_3^{9^x}}$

$fog(x) \to f(\log_3^x) \to \boxed{9^{\log_3^x}}$

$\to 9^{\log_3^x} \leqslant 9^{\log_3^x}$ (۹) $\xrightarrow{\log_3^9 = 2} x^2 \leqslant 2x$

$a^{\log_c^b} = b^{\log_c^a}$ $\quad \log_\Delta^{O^\square} = \square \log_\Delta^O$

$x^2 - 2x \leqslant 0$, $x(x-2)$ $\to$ + ۰ − ۲ + (جواب) $\to x \in [0,2]$ ✓ (۲)

(۷) خدایی این دیگه راه حل داشت ! ☺

$f\left(\frac{x^2-2}{x}\right) = \underbrace{\left(x - \frac{2}{x}\right)}_{t} + \underbrace{\left(x^2 - 4 + \frac{4}{x^2}\right)}_{t^2} \to \boxed{f(t) = t^2 + t}$ ✓

$\boxed{t = x - \frac{2}{x}}$ $\quad \boxed{t^2 \to x^2 - \frac{4}{x^2} - 4}$

(۲)

(۸)

$D_{fog} \to \{x \in D_g \mid g(x) \in D_f\}$

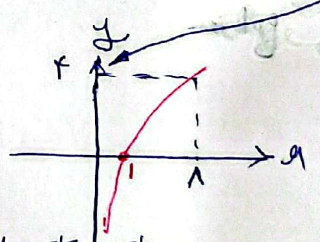
$D_g = R \to$ چون چندجمله‌ای است ✓

$D_f \Rightarrow \log_2^x \Rightarrow x > 0$

$8x - x^2 > 0$: − ۰ + ۸ − $\to \boxed{x \in (0,8)}$

$\Rightarrow D_{fog} = (0,8)$ (۱/۷۵)

برد $\to fog \to f(8x - x^2) \to \log_2^{8x-x^2} \to$ برای $x \in (0,8)$ $\to R = (0,16] \to \log_2^{16} = 4$



$R_{fog} = (-\infty, 4]$

$A \cap B \to (-\infty, 4] \cap (0,8) \to \boxed{(0,4]}$ ✓ → ۴ عدد صحیح

(۹)

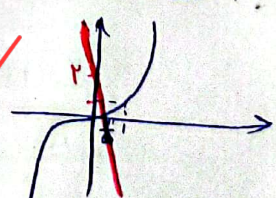
$fof(x) = 1 - 3f^2(x) \xrightarrow{x = f^{-1}(x)} f(f(f^{-1}(x))) = 1 - 3f^2(f^{-1}(x)) \to \boxed{f(x) = 1 - 3x^2}$

$x = f(f^{-1}) = f^{-1}(f)$, $1 - 3(x)^2$

$h(g) \to h(x-1) \to (x-1)^3 + 2(x-1) + 1 \to x^3 - 3x^2 + 3x + 2x - 2 + 1 \to x^3 - 3x^2 + 5x - 1$

$\to x^3 - 3x^2 + 5x - 1 = 1 - 3x^2 \to x^3 + 5x - 2 = 0 \to x^3 = -5x + 2$

شکل



یک جواب ✓ (۲)

(۱۰)

$g(2) = 8 \to g(\underbrace{f(x)}_{3}) = 8 \to f(x) = 3 \to x - \frac{4}{x} = 3 \to x^2 - 4 = 3x \to x^2 - 3x - 4$

$\frac{x^2-4}{x}$ $\quad \to (x-4)(x+1)$ (۴) (−۱)

$gof(4) = 94a + 8 = 8 \to \boxed{a = 0}$ ✓

$gof(-1) = -a - 2 = 8 \to \boxed{a = -10}$ (۲)