

①

$$f(x) = \sqrt{\log_r^{(x-1)}} \quad g(x) = \sqrt{-x^2 + 4x - 4} = \sqrt{-(x-2)^2} \quad D = \{x \in D_f \mid f(x) \in D_g\}$$

$$g(f(x)) = \sqrt{-\left(\sqrt{\log_r^{(x-1)}} - 2\right)^2} \Rightarrow -\left(\sqrt{\log_r^{(x-1)}} - 2\right)^2 \geq 0 \Rightarrow \left(\sqrt{\log_r^{(x-1)}} - 2\right)^2 \leq 0 \Rightarrow \sqrt{\log_r^{(x-1)}} - 2 = 0 \Rightarrow$$

$$\log_r^{(x-1)} = 4 \Rightarrow x-1 = r^4 = 16 \Rightarrow x = 17$$

$$D_{g \circ f} = \{17\}$$

$$\log_r^{(x-1)} \geq 0 \Rightarrow x-1 \geq 1 \Rightarrow x \geq 2$$

$$f(x) = 2x - 5 \quad g(x) = x^2 - 3x + 1$$

$$f \circ g(x) = g \circ f(x) \Rightarrow 2(x^2 - 3x + 1) = (2x - 5)^2 - 3(2x - 5) + 1 \Rightarrow$$

$$\Rightarrow 2x^2 - 6x + 2 = 4x^2 - 20x + 25 - 6x + 15 + 1 \Rightarrow 2x^2 - 6x + 2 = 4x^2 - 26x + 41 \Rightarrow 2x^2 - 20x + 39 = 0 \Rightarrow x^2 - 10x + 19.5 = 0$$

$$g \circ f(x) = 5x^2 + 11 \quad f(x) = 2x$$

$$2x = t \Rightarrow x = \frac{t}{2} \quad g(x) = 5\left(\frac{t}{2}\right)^2 + 11 = \frac{5t^2}{4} + 11 \Rightarrow$$

$$g(x-v) = \frac{5(x-v)^2}{4} + 11 = \frac{5x^2 + 2xv + v^2}{4} + 11 = \frac{5x^2 + 2xv + v^2 + 44}{4}$$

$$x = \frac{-b}{2a} = \frac{-v}{10} = v \Rightarrow v = 0$$

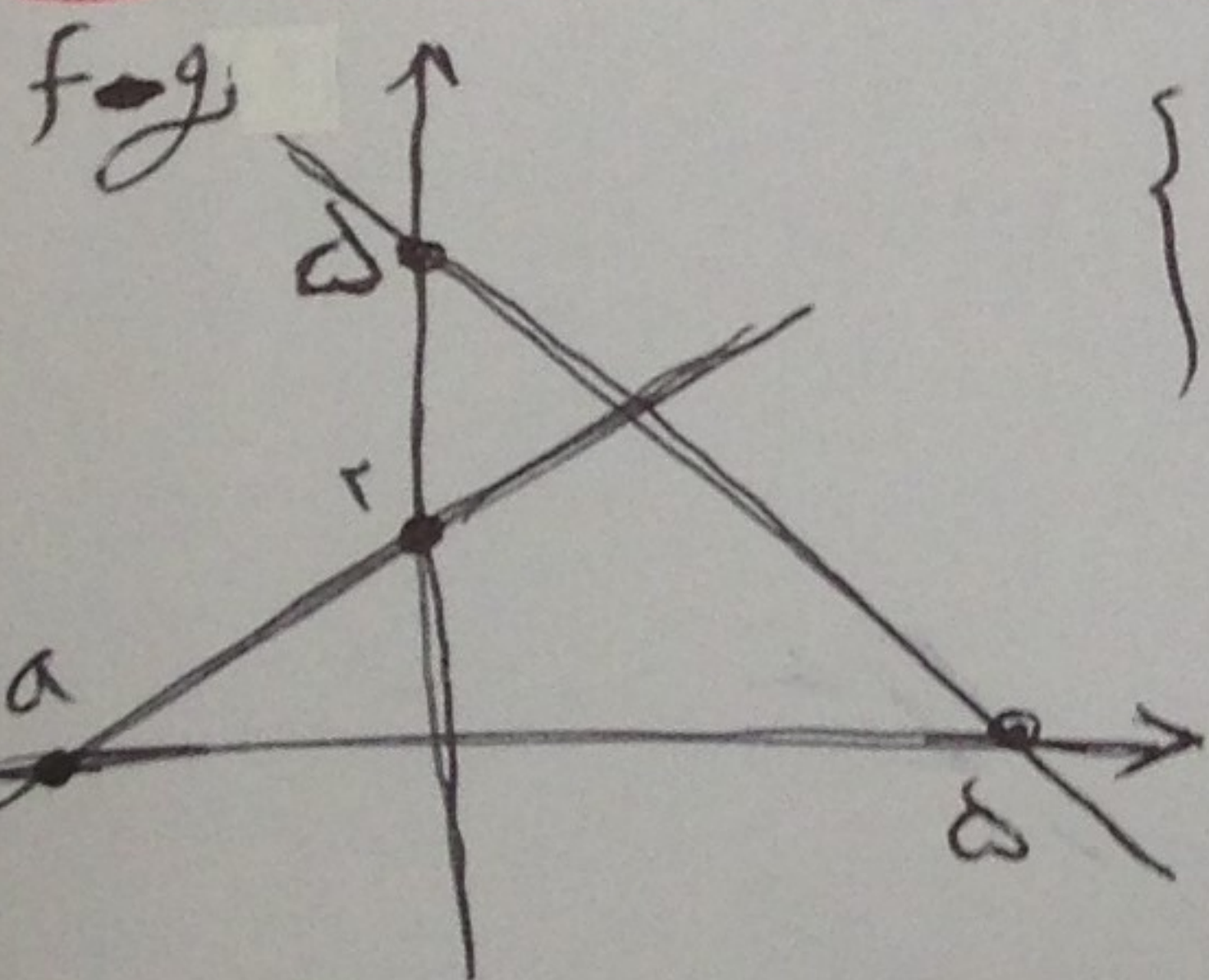
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$$f(x) = 2x - 4 \quad f \circ g(x) = 2x^2 - 4x - 1$$

④

$$f(g(x)) = 3(g(x)) - 4 = 3x^2 - 4x - 1 \Rightarrow g(x) = \frac{3x^2 - 4x - 1}{3} = x^2 - \frac{4}{3}x - \frac{1}{3}$$

$$g(f(x)) = (3x - 4)^2 - 3(3x - 4) + 1 = 9x^2 - 24x + 16 - 9x + 12 + 1 = 9x^2 - 33x + 29$$



$$\begin{cases} 2f(x) - 3g(x) = -2x + 10 \\ 3f(x) - 2g(x) = 14 \end{cases}$$

$$\Rightarrow 4f(x) = 2x + 10 \Rightarrow f(x) = \frac{1}{2}x + \frac{5}{2}$$

$$f \circ g(x) = 2(2x - 1) + 4 = 4x + 2 \Rightarrow 4(a) + 2 = 0 \Rightarrow 4a = -2 \Rightarrow a = -\frac{1}{2}$$

$$f(x) = 9^x$$

$$g(x) = \log_9^x$$

$$f \circ g(x) = 9^{\log_9^x} = x^{\log_9^9} = x^1 = x$$

$$g \circ f(x) = \log_9^{9^x} = x \log_9^9 = x$$

$$g \circ f(x) \geq f \circ g(x) \Rightarrow x \geq x^2 \Rightarrow x - x^2 \geq 0 \Rightarrow x(1-x) \geq 0$$

$$\Rightarrow \frac{0}{-1+1} \Rightarrow [0, 1]$$

$$f\left(\frac{x^2-1}{x}\right) = x^2 + x - 1 - \frac{1}{x} + \frac{x}{x^2} = x^2 + \frac{x}{x^2} - 1 + x - \frac{1}{x} =$$

$$\frac{x^2-1}{x} = x - \frac{1}{x} = t$$

$$\left(x - \frac{1}{x}\right)^2 + x - \frac{1}{x} = t^2 + t \Rightarrow f(x) = x^2 + x$$

$$f(x) = \log_2^x \quad g(x) = 11x - x^2$$

$$A = D_{f \circ g} \quad B = R_{f \circ g}$$

$$11x - x^2 > 0 \Rightarrow \frac{0}{-1+1} \Rightarrow D_{f \circ g} = A = (0, 11)$$

$$x = \frac{-b}{2a} = \frac{11}{2} = 5.5 \Rightarrow g = 11(5.5) - (5.5)^2 = 30.25 = 14 \Rightarrow R_{f \circ g} = B = (-\infty, 11]$$

$$A \cap B = (0, 11]$$

$$f \circ f(x) = 1 - 3f'(x) \quad g(x) = x-1 \quad h(x) = x^3 + 2x+1$$

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$$f \circ f(x) = 1 - 3f'(x) \Rightarrow f(x) = 1 - 3x^2$$

$$h \circ g(x) = (x-1)^3 + 2(x-1) + 1 = x^3 - 3x^2 + 2x - 1 + 2x - 2 + 1 = x^3 - 3x^2 + 4x - 2$$

$$x^3 - 3x^2 + 4x - 2 = h \circ g(x)$$

$$h \circ g(x) = f(x) \Rightarrow x^3 - 3x^2 + 4x - 2 = 1 - 3x^2 \Rightarrow x^3 + 4x - 3 = 0 \Rightarrow$$

چون درجه 3 است، پس x ها را یکبار قطع می کنند.

$$f(x) = x - \frac{4}{x}$$

$$g \circ f(x) = ax^2 + 2x$$

$$g(3) = 1$$

$$g(3) \Rightarrow x - \frac{4}{x} = 3 \Rightarrow x^2 - 4 = 3x \Rightarrow x^2 - 3x - 4 = 0 \Rightarrow$$

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$$\Rightarrow (x+1)(x-4) = 0 \Rightarrow x = \{-1, 4\} \Rightarrow \begin{cases} x = -1 \Rightarrow g(3) = 1 \Rightarrow -a - 2 = 1 \Rightarrow a = -3 \\ x = 4 \Rightarrow g(3) = 1 \Rightarrow 9a + 1 = 1 \Rightarrow a = 0 \end{cases} \Rightarrow a = \{-3, 0\}$$

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