

۱۷,۵

$$D_{g \circ f} = \{x \in D_f \mid f(x) \in D_g\}$$

$$D_f = x-1 > 0 \rightarrow x > 1 \quad \text{I} \quad \log_2^{x-1} \geq 0 \rightarrow \log_2^{x-1} \geq \log_2^1 \rightarrow x \geq 2 \quad \text{II}$$

$$D_g = \sqrt{-(x-2)^2} \geq 0 \rightarrow x \leq 2 \rightarrow \sqrt{\log_2^{x-1}} \leq 2 \xrightarrow{\text{توان ۲}} \log_2^{x-1} \leq 4$$

$$\log_2^{x-1} \leq \log_2^{16} \rightarrow x-1 \leq 16 \rightarrow x \leq 17 \quad \text{III} \quad \text{I, II, III} \rightarrow D_{g \circ f} = [2, 17]$$

$$f \circ g = f(g(x)) \rightarrow 2(x^2 - 3x + 1) - 5 = 2x^2 - 4x + 11$$

$$g \circ f = g(f(x)) \rightarrow (2x-5)^2 - 3(2x-5) + 1 = 4x^2 - 24x + 21$$

$$f \circ g = g \circ f \rightarrow 4x^2 - 24x + 21 = 2x^2 - 4x + 11 \rightarrow 2x^2 - 20x + 10 = 0$$

$$\text{خواسته سوال: } \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 25 - 2\rho = (10)^2 - 2\left(\frac{40}{2}\right) = 10 \quad \text{✓}$$

$$2x = t \rightarrow x = \frac{t}{2} \rightarrow g(t) = 5\frac{t^2}{4} + 11 \rightarrow g(x) = \frac{5x^2}{2} + 11 \quad \text{✓}$$

$$\text{خواسته سوال: } g(x-1) = \frac{5(x-1)^2}{2} + 11 \rightarrow g(x-1) = \frac{5x^2 - 10x + 16}{2}$$

$$\text{۱۵} \quad \text{رأس آن} = \text{نقطه min تابع} = \text{کمترین مقدار} \rightarrow S = \frac{-b}{2a} = 5$$

$$f \circ g = 3x^2 - 4x - 1 \rightarrow 3g(x) - 4 = 3x^2 - 4x - 1 \rightarrow g(x) = x^2 - 2x + 1$$

$$\text{خواسته سوال: } g \circ f = g(f(x)) = (3x-4)^2 - 2(3x-4) + 1 = 9x^2 - 24x + 14 - 6x + 8 + 1 = 9x^2 - 30x + 23$$

$$g \circ f = 9x^2 - 30x + 23 \quad \text{✓} \quad \text{۲}$$

$$2g(x) = 3f(x) - 14 \rightarrow 2(f(x) - g(x)) = -f(x) + 14 \rightarrow f - g = \frac{-f(x)}{2} + 7$$

$$f(x) = ax + b, \quad \frac{-ax + b}{2} + 7 = y \rightarrow \begin{cases} (0, 5) \rightarrow b = +14 \\ (5, 0) \rightarrow a = 2 \end{cases} \rightarrow f(x) = 2x + 14$$

$$2g(x) = 3(2x + 14) - 14 \rightarrow g(x) = 3x + 14$$

$$f(g(x)) = 2(3x + 14) + 14 = 6x + 42 \quad (a, 0) \rightarrow 6a = -42 \rightarrow a = -7 \quad \text{✓}$$

$$f \circ g = f(g(x)) = 9^{\log_3 x} = x^2 \quad g \circ f = g(f(x)) = \log_3 9^x = 2x$$

$$x^2 \leq 2x \rightarrow x(x-2) \leq 0 \rightarrow \begin{array}{c} x \\ | \\ y \end{array} \begin{array}{c} 0 \\ | \\ 0 \end{array} \begin{array}{c} 2 \\ | \\ 0 \end{array} \begin{array}{c} + \\ | \\ + \end{array} \rightarrow x \in [0, 2]$$

تعداد اعضای صبیح مجموعه: ۳ عدد

$$f\left(\frac{x^2-1}{x}\right) = f\left(x - \frac{1}{x}\right) = x - \frac{1}{x} + x^2 + \frac{\varepsilon}{x^2} - \varepsilon$$

$$f\left(x - \frac{1}{x}\right) = x - \frac{1}{x} + \left(x - \frac{1}{x}\right)^2 \rightarrow f(x) = x + x^2$$

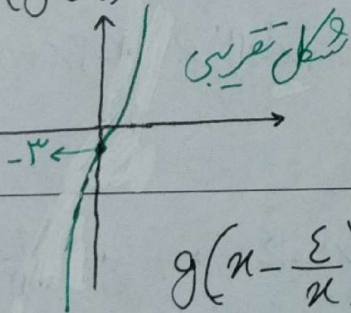
$$f \circ g = f(g(x)) \rightarrow \log_2(\wedge x - x^2) \rightarrow D_{f \circ g} = \wedge x - x^2 > 0 \rightarrow \begin{array}{c} x \\ | \\ y \end{array} \begin{array}{c} 0 \\ | \\ 0 \end{array} \begin{array}{c} \wedge \\ | \\ 0 \end{array} \begin{array}{c} - \\ | \\ - \end{array}$$

$$R_{f \circ g} = \text{شکل تقریبی} \rightarrow y \in \mathbb{R} \rightarrow R = (-\infty, \frac{1}{4}]$$

$$A \cap B = (0, 1) \cap \mathbb{R} = (0, 1) \quad A \cap B = [0, \frac{1}{4}] \rightarrow \text{عدد صبیح}$$

$$f(f(x)) = 1 - 3f^2(x) \rightarrow f(x) = 1 - 3x^2$$

$$h(g(x)) = f(x) \rightarrow (x-1)^3 + 2(x-1) + 1 = 1 - 3x^2 \rightarrow x^3 + 2x - 3 = 0$$



تعداد ریشه های آن یکی است.

$$g\left(x - \frac{\varepsilon}{x}\right) = ax^3 + 2x \begin{cases} x = \varepsilon \rightarrow g(\varepsilon) = 1 \rightarrow a \times \varepsilon^3 + 2 = 1 \\ x = -1 \rightarrow g(-1) = 1 \rightarrow -a - 2 = 1 \end{cases}$$

$$x - \frac{\varepsilon}{x} = 1 \rightarrow x^2 - 3x - \varepsilon = 0 \quad (x - \varepsilon)(x + 1) = 0$$