

Subject: _____

Date: _____

$$f(n+2) = \begin{cases} 3n+5 & n \geq 1 \\ 2n-1 & n < 1 \end{cases} \Rightarrow \begin{cases} 3(n+2)-1 & n \geq 1 \\ 2(n+2)-9 & n < 1 \end{cases} \rightarrow f(n) = \begin{cases} 3n-1 & n \geq 1 \\ 2n-9 & n < 1 \end{cases} \quad (1)$$

$$f(3-n) \geq f(n+1) \xrightarrow{\text{تابع صعودی}} 3-n \geq n+1 \Rightarrow \boxed{n \leq 1} \quad \checkmark \quad (2)$$

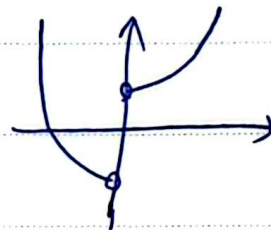
$$f \circ f(n) \leq f(n^3) \xrightarrow{\text{تابع اکید صعودی}} f(n) \leq n^3 \rightarrow (n^3+n)^3 \leq n^{\frac{3}{2}} \rightarrow n^3+n \leq n^{\frac{3}{2}} \quad (2) \quad 5$$

$$\checkmark \quad \boxed{n \leq 0} \leftarrow n^3 \leq 0 \quad \checkmark$$

$$y = |x| \left(x^2 + \frac{1}{x} \right) \rightarrow x \neq 0$$

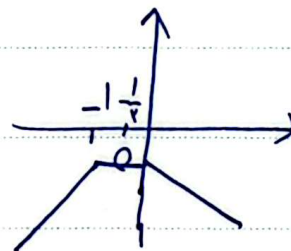
$$x > 0 \rightarrow x \left(x^2 + \frac{1}{x} \right) = x^3 + 1$$

$$x < 0 \rightarrow -x \left(x^2 + \frac{1}{x} \right) = -x^3 - 1$$



(3) تابع غیر یکنوا $\checkmark$

$$\begin{cases} x > 0 \rightarrow \frac{2n+1}{-1} = -2n-1 & \text{نزولی} \\ 0 < x < -1 \rightarrow \frac{2n+1}{-2n-1} = -1 & \text{ثابت} \\ x < -1 \rightarrow \frac{2n+1}{-n-n+1} = 2n+1 & \text{صعودی} \end{cases}$$



(4) $\boxed{a_{n+1} = 0} \quad \checkmark$

$$2n^2 - 2n - 1 > 0 \Rightarrow (n-1)(n+2) > 0 \xrightarrow{+1-1} (-\infty, -2) \cup (2, +\infty) \quad (5)$$

$$n_0 = \frac{1}{2} \rightarrow \text{بین صفر و ۱} \Rightarrow \text{و اما کافی} \xrightarrow{*} \boxed{(-\infty, -2)} \quad \checkmark \Rightarrow \text{اکید صعودی}$$

$$y = a^n \rightarrow a > 0 \rightarrow f(n) = (a^2-3)^n \rightarrow a^2-3 > 0 \rightarrow a > \sqrt{3} \quad (6)$$

$$+^n = (a^2-3)^n \xrightarrow{\text{تبدیل}} + > 1 \rightarrow a^2-3 > 1 \Rightarrow a > 2 \quad (2)$$

الف) $a \in (-\infty, -2) \cup (2, +\infty)$ اکید صعودی : $n_1 < n_2 \Rightarrow f(n_1) < f(n_2)$

ب) $a \in (-\infty, -2] \cup [2, +\infty)$ صعودی : $n_1 < n_2 \Rightarrow f(n_1) \leq f(n_2)$

الف) $\Rightarrow \boxed{a > 2 \vee a < -2} \quad \checkmark$

ب) $\Rightarrow \boxed{a \in (-\infty, -2] \cup [2, +\infty) \cup [-\sqrt{3}, \sqrt{3}]} \quad \checkmark$

$$f(m) = m^3 + am^2 + bm + c \xrightarrow{\text{نقطة}} f'(m) = 3m^2 + 2am + b$$

$$f(-3) = -27 + 9a - 3b + c = -1 \rightarrow 9a - 3b + c = 26$$

$$f'(-3) = 9 \times 9 + 2a(-3) + b \rightarrow 27 - 6a + b = 0 \rightarrow b = 6a - 27$$

$$\rightarrow 9a - 3(6a - 27) + c = 26 \rightarrow c = 9a - 50$$

$$f(m) = m^3 + am^2 \rightarrow (9a - 27)m + 9a - 50 \rightarrow f'(m) = 3m^2 + 9(m - \frac{9a}{3})$$

5

$$f \circ f(m) \text{ و } f(\sqrt{m}) \Rightarrow m + \sqrt{m} - 2 + \sqrt{m + \sqrt{m} - 2} \rightarrow \sqrt{m} + \sqrt{m} \leftarrow \text{نقطة}$$

$$\cos^2 m \rightarrow 1 \rightarrow \frac{1}{\sqrt{9-1}} - \frac{1}{\sqrt{1-9}} = \frac{1}{2} - \frac{1}{-2} = \frac{1}{2} + \frac{1}{2} = 1 = b$$

$$\cos^2 m \rightarrow 0 \rightarrow \frac{1}{\sqrt{9-1}} - \frac{1}{\sqrt{1-9}} = \frac{1}{2} - \frac{1}{-2} = \frac{1}{2} + \frac{1}{2} = 1 = b - a = \frac{10}{2} \left(\frac{1}{2} \right) = \frac{10}{2}$$

$$f(m) = m - [m+2] \quad g(m) = \frac{2^m - 2^{-m}}{2} \quad g(f(m)) = \frac{2^{m-[m+2]} - 2^{-m+[m+2]}}{2}$$

$$R_f = [-2, 9-1] \Rightarrow g(2) = \frac{2^2 - 2^{-2}}{2} = \frac{1}{2}$$

$$g(-1) = \frac{2^{-1} - 2^1}{2} = \frac{1}{2} - 2 = -\frac{3}{2}$$

$$R = \left[\frac{1}{2}, -\frac{3}{2} \right]$$