

$$\begin{aligned} \text{الف)} \quad y'' - 2y' &= y - 1 \longrightarrow y_1'' - 2y_1' = y_1 - 1 \implies y_1'' - 2y_1' = y_2'' - 2y_2' \\ &\implies y_1'' - 2y_1' = y_2'' - 2y_2' \\ &\implies y_1'' - y_2'' = 2y_1' - 2y_2' \\ &\implies (y_1 - y_2)'' = 2(y_1 - y_2)' \\ &\implies (y_1 - y_2)'' - 2(y_1 - y_2)' = 0 \\ &\implies (y_1 - y_2)' = 0 \implies y_1 - y_2 = 0 \implies y_1 = y_2 \end{aligned}$$

تابع است $\iff$ همراه مثبت است و در حالت $y_1 = y_2$ صفری شود

$$\text{ب)} \quad x \cos y = y \cos x \implies \frac{x}{y} \cos y = \cos x \implies \frac{x}{y} \cos y = 0 \implies y = \left(\frac{\pi}{2} + k\pi \right)$$

مثال نقض

$$\text{ج)} \quad \frac{x+y}{y} = \sqrt{xy} \implies x+y - \sqrt{xy} = 0 \implies (\sqrt{x} + \sqrt{y})^2 = 0 \implies x=y \implies \text{تابع است}$$

مثال نقض

$$\text{د)} \quad xy^2 + 2x - y - 2y = 0 \implies x(y^2 + 2) - y(y + 2) = 0 \implies (x-y)(y^2 + 2) = 0 \implies x=y \implies \text{تابع است}$$

تابع نیست

$$\text{ب)} \quad [y] + [a] = 1 \implies [y] + 0 = 1 \implies y = [1 \quad 2]$$

تابع است

$$\text{الف)} \quad |y| + x^2 = 2y \implies y + x^2 = 2y \implies x^2 = y \implies y > 0$$

تابع نیست

$$y < 0 \implies x^2 = y \implies y = \frac{x^2}{2} \implies \text{تابع نیست}$$

تابع نیست

$$f(x) = \begin{cases} x^2 - b & |x-1| \geq 2 \\ a + 2x & 0 < x < 3 \end{cases} \implies x=2 \implies f(2) - b = a + 2(2) \implies 1x = b+a$$

تابع نیست

$$a - ab - a^2 = 0 \implies a(1-b-a) = 0 \implies \begin{cases} a=0 \\ b+a=1 \\ a=-b+1 \end{cases} \implies \begin{cases} -1=b & a=1 \\ \frac{1}{y}=b & a=\frac{1}{y} \end{cases} \implies 1 = a+b$$

$$y = \frac{r}{a^2 + b^2 + c^2}$$

$$ra + rb + c = 0$$

$$b < 0$$

-d

$$a = 0 \Rightarrow \text{ت/ب}$$

$$\Rightarrow rb + c = 0$$

$$c = -rb$$

$$ra + c - r(b) \Rightarrow -rb + rb = \boxed{b}$$

$$y = \sqrt{\frac{ra+r}{a+b}}$$

$$r+b=0$$

$$\boxed{b=-r}$$

-f

$$y = \sqrt{a^2 - x} \quad x > a, \quad a(a-1)(a+1). \quad D_f = [-1, 0] \cup [1, +\infty)$$

$$\begin{array}{ccc} -1 & 0 & 1 \\ -1 & + & 1 \\ -1 & + & 1 \end{array}$$

-v

$$y = \frac{1}{\sqrt{|x|-1}}$$

$$|x| > 1$$

$$\begin{array}{ccc} -1 & 1 \\ + & - \\ + & - \end{array}$$

$$D_f = (-\infty, -1) \cup (1, +\infty)$$

$$y = \sqrt{\frac{1}{|x|-2}}$$

$$|x| - [2] \neq 0$$

$$x \neq \pm 2$$

$$\sqrt{x} + \sqrt{y} = 9 \quad x > 0$$

-A

$$D_f = \mathbb{R} - \{ \pm 2 \}$$

$$9 - \sqrt{x} > 0 \Rightarrow x < 81$$

$$D_f = [0, 81]$$

$$y = \frac{1}{\left(x - \frac{1}{r}\right) + \left(x + \frac{r}{r}\right)}$$

$$\left(x - \frac{1}{r}\right) + \frac{r}{r} \left(x + \frac{r}{r}\right) > 0 \Rightarrow \left(x - \frac{1}{r}\right) + \left(x + \frac{r}{r} - \frac{r}{r}\right) > 0$$

-9

$$r \left(x - \frac{1}{r}\right) + 1 > 0$$

$$r \left(x - \frac{1}{r}\right) > -1$$

$$\left(x - \frac{1}{r}\right) > -\frac{1}{r}$$

$$x - \frac{1}{r} > 0$$

$$\boxed{x > \frac{1}{r}}$$

$$D_f = \left[\frac{1}{r}, +\infty\right)$$

$$y = \sqrt{1 - \log(x-a)}$$

$$\log(x-a) < 1$$

$$\Rightarrow x-a < 10$$

$$x-a < 10$$

$$x-b < \frac{10}{r}$$

$$c = \frac{10+a}{r}$$

$$b = \frac{a}{r}$$

$$c-b = \frac{10+a - \frac{a}{r}}{r} = \boxed{d}$$

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