

$$L(r) - L(-1) \Rightarrow L(r) \Rightarrow 1 - r_n = r \Rightarrow n = -1 \Rightarrow L(r) = \sqrt[3]{-1} + r = 1$$

$$\downarrow$$

$$L(-1) \Rightarrow 1 - r_n = -1 \Rightarrow -r_n = -r \Rightarrow n = 1 \Rightarrow L(-1) = \sqrt[3]{-1} = -1$$

$$1 + r = 0$$

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$$L(n + \sqrt{n}) = n^{\frac{1}{3}} + r_n \Rightarrow n > 0$$

$$L(4) \Rightarrow n + \sqrt{n} = 4 \xrightarrow{\sqrt{n}=t} t^2 + t - 4 = 0 \Rightarrow (t+4)(t-1) = 0$$

$$\begin{cases} t_1 = 4 \checkmark \rightarrow \sqrt{n} = 4 \Rightarrow n = 16 \\ t_2 = -1 \text{ لا يجوز } \end{cases}$$

$$\xrightarrow{n=16} L(4) \cdot (16)^{\frac{1}{3}} + r(16) = 4 \cdot 4 = 16$$

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$$L(n) = n^{\frac{1}{3}} + n \quad L(n-1) = (n-1)^{\frac{1}{3}} + (n-1)$$

$$\Rightarrow L(1-n) = (1-n)^{\frac{1}{3}} + (1-n) \Rightarrow -(n-1)^{\frac{1}{3}} - (n-1)$$

$$L(n-1) + L(1-n)$$

$$\downarrow$$

$$(n-1)^{\frac{1}{3}} + (n-1) - (n-1)^{\frac{1}{3}} - (n-1) = 0$$

$$L(n-1) + L(1-n) = 0$$

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$$D_{f \cdot g} = D_f \cap D_g \rightarrow D_f = \{n - \frac{1}{f} \mid n \in \mathbb{Z}\} \Rightarrow n \geq \frac{1}{f} \Rightarrow D_f \cap D_g = \left[\frac{1}{f}, \frac{b}{f} \right]$$

$$D_g = \{b - \frac{1}{g} \mid n \in \mathbb{Z}\} \Rightarrow n \leq \frac{b}{f}$$

$$f \cdot g = \{ (1, k) \}$$

$$\downarrow$$

$$(f \cdot g)(1) = f(1) \cdot g(1) = k \Rightarrow \sqrt[3]{1} \cdot \sqrt[3]{1} = k \Rightarrow 1 \cdot 1 = k \Rightarrow k = 1$$

$$n=1 \Rightarrow \frac{1}{f} = 1 \Rightarrow a = \frac{1}{f}, \quad \frac{b}{f} = 1 \Rightarrow b = f \quad \left| \frac{1}{a} - \frac{b}{f} = k \Rightarrow \frac{f}{f} - \frac{f}{f} = 0 \right.$$

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$$D_{f \cdot g} = D_f \cap D_g \rightarrow D_g = \{m + 1 \mid m \in \mathbb{Z}\} \Rightarrow m \geq -1$$

$$D_f = \{m - n \mid m \in \mathbb{Z}\} \Rightarrow m \geq n$$

$$\Rightarrow D_g \cap D_f = [-1, m] = [-1, v] \Rightarrow m = v$$

$$(f \cdot g)(r) = f(r) \cdot g(r) = r \sqrt{r} \Rightarrow r + n - r \sqrt{r} = r \sqrt{r} \Rightarrow n = 2r \sqrt{r}$$

$$\sqrt{r-4} + n = r + n \quad \sqrt{r(r)+r} = \sqrt{r}$$

$$m+n = v + \sqrt{r-4} = 0 + \sqrt{r-4}$$

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