

$$[x] > 4 \Rightarrow x > 5 \rightarrow y = \sqrt{2x-1-x+1} \Rightarrow y = \sqrt{x} \quad x > 5 \quad \checkmark$$

$$[x] \leq 4 \Rightarrow x < 5 \rightarrow y = \sqrt{4x+3-x+1} \Rightarrow y = \sqrt{3x+4} \Rightarrow x > -\frac{4}{3} \Rightarrow \checkmark$$

$$D_y = \left[-\frac{4}{3}, +\infty\right) \quad \checkmark$$

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$$\frac{24x^3+36x^2+18x+3}{(4x^2+4x+1)^2} = \frac{3(8x^3+12x^2+6x+1)}{(2x+1)^4} = \frac{3(2x+1)^3}{(2x+1)^4} = \frac{3}{2x+1}$$

$$\Rightarrow 2x+1 \neq 0 \Rightarrow x \neq -\frac{1}{2}$$

$$D_f = \mathbb{R} - \left\{-\frac{1}{2}\right\}$$

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$$\left. \begin{aligned} f(0) = \frac{-2}{-1} = 2 &\Rightarrow f(0) = g(0) \Rightarrow g(0) = 2 \\ &\Rightarrow c = 2 \quad \checkmark \\ g(1) = f(1) &\Rightarrow 3 = \frac{a+2}{b+1} \\ g(-1) = f(-1) &\Rightarrow 1 = \frac{2-a}{b-1} \end{aligned} \right\} \begin{aligned} 3b+3 &= a+2 \\ b-1 &= 2-a \end{aligned} \Rightarrow \begin{aligned} 4b+2 &= 4 \\ b &= \frac{1}{2} \quad \checkmark \end{aligned}$$

$$b+c = 2 + \frac{1}{2} = \frac{5}{2}$$

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$$\frac{x+1}{x-c} = \frac{(x+1)(x-1)}{x^2+ax+b} \Rightarrow (x-1)(x-c) = x^2+ax+b \Rightarrow x^2-(c+1)x+c = x^2+ax+b \Rightarrow b=c$$

$$D_f = \mathbb{R} - \{c\} \rightarrow \text{پس } g \text{ هم باید یک ریشه منفرجه داشته باشد} \rightarrow x^2+ax+b = (x-1)^2$$

$$\Rightarrow c = 1 \Rightarrow a+b+c = -c-1+c+c = -1+1+1 = 1$$

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$$f(x+1) = (x-2)^2(x) = x(x^2-4x+4) = x^3-4x^2+4x$$

$$f(1-x) = (-x-2)^2(-x) = -x(x^2+4x+4) = -x^3-4x^2-4x$$

$$f(1-x) + f(x+1) = x^3-4x^2+4x - x^3-4x^2-4x$$

$$\Rightarrow g(x) = -8x^2 \quad \checkmark$$

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$\left. \begin{aligned} 1-2x = -1 &\Rightarrow x = 1 \Rightarrow f(-1) = \sqrt[4]{1} = 1 \\ 1-2x = 3 &\Rightarrow x = -1 \Rightarrow f(3) = \sqrt[3]{-1} + 2 = 1 \end{aligned} \right\} f(-1) + f(3) = 1 + 1 = 2$	(2) 6
$x + \sqrt{x} = 6 \xrightarrow{\sqrt{x}=t} t^2 + t - 6 = 0 \Rightarrow t < \frac{-3}{2} \times \Rightarrow \sqrt{x} = 2 \rightarrow x = 4$ $\Rightarrow f(6) = 4^4 + 3(4) = 256 + 12 = 268$	(2) 7
$f(x-1) + f(1-x) = f(t) + f(-t)$ $\Rightarrow \left. \begin{aligned} f(t) &= t^3 + t \\ f(-t) &= -t^3 - t \end{aligned} \right\} f(t) + f(-t) = t^3 + t - t^3 - t = 0$ $\Rightarrow f(x-1) + f(1-x) = 0$	(2) 8
$\left. \begin{aligned} b-4x > 0 &\Rightarrow x < \frac{b}{4} \\ 4x-3a > 0 &\Rightarrow x > \frac{3a}{4} \end{aligned} \right\} \frac{3a}{4} < x < \frac{b}{4} \rightarrow$ در دامنه فقط $x=1$ هست سه هم $\frac{b}{4}$ و هم $\frac{3a}{4}$ برابر می شوند $\rightarrow \frac{b}{4} = 1 \Rightarrow b = 4$ $\rightarrow \frac{3a}{4} = 1 \Rightarrow a = \frac{4}{3}$ $3a - \frac{b}{2} - k = 4 - 2 + 1 = 3$ $2f(1) - g(1) = k \rightarrow 2(\sqrt{4-4} + k - 1) - (\sqrt{4-4} + 3k) = k$ $\rightarrow 2k - 2 - 3k = k \Rightarrow k = -1$	(2) 9
$\left. \begin{aligned} 2x+2 > 0 &\Rightarrow x > -1 \\ m-x > 0 &\Rightarrow x < m \end{aligned} \right\} D_{f \times g} = [-1, m] \xrightarrow{\text{لنفه سوال [3 دان]}} m = 7$ $f(3) - g(3) = 6\sqrt{2} \Rightarrow \sqrt{7-3} + n - 2\sqrt{2} = 6\sqrt{2}$ $\Rightarrow 2 + n = 8\sqrt{2} \Rightarrow n = 8\sqrt{2} - 2$ $\Rightarrow m + n = 7 + 8\sqrt{2} - 2 = 5 + 8\sqrt{2}$	(2) 10