

$$[x] > 4 \Rightarrow x > 5 \rightarrow y = \sqrt{2x-1-x+1} \Rightarrow y = \sqrt{x} \quad x > 5 \quad \checkmark$$

$$[x] \leq 4 \Rightarrow x < 5 \rightarrow y = \sqrt{4x+3-x+1} \Rightarrow y = \sqrt{3x+4} \Rightarrow x > -\frac{4}{3} \Rightarrow \checkmark$$

$$D_y = \left[-\frac{4}{3}, +\infty\right)$$

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$$\frac{24x^3 + 36x^2 + 18x + 3}{(4x^2 + 4x + 1)^2} = \frac{3(8x^3 + 12x^2 + 6x + 1)}{(2x+1)^4} = \frac{3(2x+1)^3}{(2x+1)^4} = \frac{3}{2x+1}$$

$$\Rightarrow 2x+1 \neq 0 \Rightarrow x \neq -\frac{1}{2}$$

$$D_f = \mathbb{R} - \left\{-\frac{1}{2}\right\}$$

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$$\left. \begin{array}{l} f(0) = \frac{-2}{-1} = 2 \Rightarrow f(0) = g(0) \Rightarrow g(0) = 2 \\ \Rightarrow c = 2 \\ g(1) = f(1) \Rightarrow 3 = \frac{a+2}{b+1} \\ g(-1) = f(-1) \Rightarrow 1 = \frac{2-a}{b-1} \end{array} \right\} \begin{array}{l} 3b+3 = a+2 \\ b-1 = 2-a \end{array} \Rightarrow \begin{array}{l} 4b+2 = 4 \\ b = \frac{1}{2} \end{array} \left. \vphantom{\begin{array}{l} f(0) = \frac{-2}{-1} = 2 \\ g(1) = f(1) \Rightarrow 3 = \frac{a+2}{b+1} \\ g(-1) = f(-1) \Rightarrow 1 = \frac{2-a}{b-1} \end{array}} \right\} b+c = 2+\frac{1}{2} = \frac{5}{2}$$

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$$\frac{x+1}{x-c} = \frac{(x+1)(x-1)}{x^2+ax+b} \Rightarrow (x-1)(x-c) = x^2+ax+b \quad a = -c-1$$

$$\Rightarrow x^2 - (c+1)x + c = x^2 + ax + b \Rightarrow b = c$$

$$D_f = \mathbb{R} - \{c\} \rightarrow \text{پس } g \text{ هم باید یک ریشه منفرجه داشته باشد} \rightarrow x^2+ax+b = (x-1)^2$$

ریشه ها ۱ باشد که با صورت سازگار شود

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$$\Rightarrow c=1 \Rightarrow a+b+c = -c-1+c+c = \cancel{-1} + \cancel{1} + \cancel{1} = \underline{0}$$

$$f(x+1) = (x-2)^2(x) = x(x^2-4x+4) = x^3-4x^2+4x$$

$$f(1-x) = (-x-2)^2(-x) = -x(x^2+4x+4) = -x^3-4x^2-4x$$

$$f(1-x) + f(x+1) = x^3-4x^2+4x - x^3-4x^2-4x$$

$$\Rightarrow g(x) = -8x^2$$

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$\left. \begin{aligned} 1-2x = -1 &\Rightarrow x = 1 \Rightarrow f(-1) = 4\sqrt{1} = 4 \\ 1-2x = 3 &\Rightarrow x = -1 \Rightarrow f(3) = 3\sqrt{-1} + 2 = 1 \end{aligned} \right\} f(-1) + f(3) = 4 + 1 = 5$	6
$x + \sqrt{x} = 6 \xrightarrow{\sqrt{x}=t} t^2 + t - 6 = 0 \Rightarrow t < \frac{-3}{2} \times \Rightarrow \sqrt{x} = 2 \rightarrow x = 4$ $\Rightarrow f(6) = 4^4 + 3(4) = 256 + 12 = 268$	7
$f(x-1) + f(1-x) = f(t) + f(-t)$ $\Rightarrow \left. \begin{aligned} f(t) &= t^3 + t \\ f(-t) &= -t^3 - t \end{aligned} \right\} f(t) + f(-t) = t^3 + t - t^3 - t = 0$ $\Rightarrow f(x-1) + f(1-x) = 0$	8
$\left. \begin{aligned} b-4x > 0 &\Rightarrow x < \frac{b}{4} \\ 4x-3a > 0 &\Rightarrow x > \frac{3}{4}a \end{aligned} \right\} \frac{3}{4}a < x < \frac{b}{4} \rightarrow$ در دامنه فقط $x=1$ هست پس هم $\frac{b}{4}$ و هم $\frac{3}{4}a$ برابر 1 شوند $\rightarrow \frac{b}{4} = 1 \Rightarrow \underline{b=4}$ $\rightarrow \frac{3}{4}a = 1 \Rightarrow \underline{a=\frac{4}{3}}$ $3a - \frac{b}{2} - k = 4 - 2 + 1 = 3$ $\rightarrow 2k - 2 - 3k = k \Rightarrow \underline{k=-1}$	9
$\left. \begin{aligned} 2x+2 > 0 &\Rightarrow x > -1 \\ m-x > 0 &\Rightarrow x < m \end{aligned} \right\} D_{f \times g} = [-1, m] \xrightarrow{\text{لنفه سوال [7 دان]}} m = 7$ $f(3) - g(3) = 6\sqrt{2} \Rightarrow \sqrt{7-3} + n - 2\sqrt{2} = 6\sqrt{2}$ $\Rightarrow 2 + n = 8\sqrt{2} \Rightarrow n = 8\sqrt{2} - 2$ $\Rightarrow m + n = 7 + 8\sqrt{2} - 2 = 5 + 8\sqrt{2}$	10