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$$[0, \infty) \cap [0, \infty) = [0, \infty)$$

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$$f(x) = x + 1 \geq 0 \quad \underline{x \geq 0}, \quad x - 1 - x + 1 \geq 0 \rightarrow x \geq 0$$

$$\rightarrow \underline{x < 0}, \quad x + 1 - x + 1 \geq 0 \quad x + 1 \geq 0 \rightarrow x \geq -1 \quad (-\infty, 0) \cap [-1, \infty)$$

$$[-1, 0) \cup [0, \infty) = [-1, \infty)$$

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$$x^p + y^p + 1 \mid x + y = p \mid x^p + 1 \mid y + 1 = p \mid x + y + 1 = p \mid x + y + 1$$

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$$x^p + y^p + 1 = (x + y)^p \rightarrow (x^p + y^p + 1) \leq (x + y)^p$$

$$\frac{x^p + y^p + 1}{(x + y)^p} \leq \frac{1}{x + y} \quad D \in (-\infty, -\frac{1}{p}) \cup (-\frac{1}{p}, \infty)$$

$$\frac{x^p + y^p - x - y}{x^p - 1} = \frac{(x + y)(x^{p-1} - 1)}{x^p - 1} \leq x + y \quad f(x) \in \mathbb{Q}(x) \rightarrow x + y \leq x + y \Rightarrow x \leq y$$

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$$f(1) \leq \frac{a + y}{1 + b} \leq 1 \rightarrow a + y \leq 1 + b \rightarrow a = 1 + b \rightarrow b + 1 \leq 1 + b \rightarrow b \leq 1$$

$$f(-1) = \frac{-a + y}{b - 1} \leq 1 \rightarrow -a + y \leq b - 1 \rightarrow -a = b - 1 - y \rightarrow a = 1 - b$$

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$$\frac{x + 1}{x - 1} = \frac{x^p - 1}{x^p + a x + b} \Rightarrow (x + 1)(x^p + a x + b) = (x - 1)(x^p - 1) \Rightarrow$$

$$x^p + a x^p + b x + x + 1 = x^p - 1 + a x^p + a x + b = x^p - 1 + a x^p - x + 1$$

$$\Rightarrow b \leq 1 \quad a + 1 = 1 \rightarrow a \leq -1 \quad a + b \leq -1$$

$$(2, 2) \rightarrow c = 1, b$$

$$\frac{x + 1}{x - 1} \leq \frac{x^p - 1}{x^p - (x + 1)x + c} \Rightarrow \frac{x^p - 1}{x^p - (x + 1)x + c} \leq \frac{(x - 1)(x + 1)}{(x - 1)(x - c)} \Rightarrow$$

$$a \leq -1$$

$$a + b + c \leq -1 + 1 + 1 = 1$$

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$$f_{(n+1)} \leq ((n+1) - 1)^p \times ((n+1) - 1) = (n - 1)^p (n) = n(n - 1)^p$$

$$f_{(1-n)} \leq ((1-n) - 1)^p ((1-n) - 1) = (-n - 1)^p (-n) \Rightarrow -n(n + 1)^p$$

$$f_{(n+1)} + f_{(1-n)} \leq n(n - 1)^p + -n(n + 1)^p \Rightarrow n((n - 1)^p - (n + 1)^p) \Rightarrow$$

$$\Rightarrow n(x^p + 1 - (x - x^p - 1 - x)) \leq n(-1 - x) = -1 - x$$

$1 - \sqrt{1-x} = \sqrt{x} \rightarrow -\sqrt{1-x} + \sqrt{x} \rightarrow x = -1 \Rightarrow f_{(-1)} = \sqrt{-1+1} = 0$ $1 - \sqrt{1-x} = -1 \Rightarrow -\sqrt{1-x} = -2 \Rightarrow \sqrt{1-x} = 2 \Rightarrow 1-x = 4 \Rightarrow x = -3 \Rightarrow f_{(-3)} = \sqrt{-3+1} = \sqrt{-2}$ $f_{(1)} + f_{(-1)} = 1 + 0 = 1$	(2) 6
$x + \sqrt{x} = 4 \rightarrow t^2 + t - 3 = 0 \Rightarrow (t-1)(t+3) = 0 \begin{cases} t=1 \\ t=-3 \end{cases}$ $\sqrt{x} = 1 \rightarrow x = 1$ $\sqrt{x} = -3 \rightarrow x = 9$ $f(4) = f(1) + f(9) = 1 + 3 = 4$	(2) 7
$f_{(n-1)} = (n-1)^n + (n-1) = n^n - n^{n-1} + n - 1 + n - 1 = n^n - n^{n-1} + 2n - 2$ $f_{(1-n)} = (1-n)^n + (1-n) = 1 - n^n + n^{n-1} - n + 1 - n = 2 - n^n + n^{n-1} - 2n$ $f_{(1-n)} + f_{(n-1)} = 2 - n^n + n^{n-1} - 2n + n^n - n^{n-1} + 2n - 2 = 0$	(2) 8
$\sqrt{1-x} + (x-1) = 0 \Rightarrow \sqrt{1-x} = 1-x \Rightarrow 1-x = (1-x)^2 \Rightarrow 1-x = 1-2x+x^2 \Rightarrow x^2 - x = 0 \Rightarrow x(x-1) = 0 \Rightarrow x = 0 \text{ or } x = 1$ $f_{(0)} = \sqrt{1-0} + (0-1) = 1-1 = 0$ $f_{(1)} = \sqrt{1-1} + (1-1) = 0+0 = 0$ $f_{(0)} + f_{(1)} = 0+0 = 0$	(2) 9
$\sqrt{m-x} + x = 0 \Rightarrow \sqrt{m-x} = -x \Rightarrow m-x = x^2 \Rightarrow x^2 + x - m = 0$ $\Delta = 1 + 4m \geq 0 \Rightarrow m \geq -\frac{1}{4}$ $x = \frac{-1 \pm \sqrt{1+4m}}{2}$ $f_{(x)} = \sqrt{m-x} + x = -x + x = 0$	(2) 10