

$$f(x) = x+1 \geq 0 \xrightarrow{x \geq 0} x+1 \geq 0 \rightarrow x \geq 0 \quad [0, \infty) \cap [0, \infty) = [0, \infty)$$

$$\rightarrow \xrightarrow{x < 0} x+1 \geq 0 \quad x+1 \geq 0 \rightarrow x \geq -1 \quad (-\infty, 0) \cap [-1, \infty) = [-1, 0)$$

$$[-1, 0) \cup [0, \infty) = [-1, \infty)$$

1

2

3

4

5

$$x^p + x^p + 1 \wedge x+1 = x^p (x^p + 1) + x+1 = x^p (x^p + 1) + x+1 = x^p (x^p + 1) + x+1$$

$$x^p + x+1 = (x+1)^p \rightarrow (x^p + x+1) \leq (x+1)^p$$

$$\frac{x^p (x^p + 1)}{(x+1)^p} \leq \frac{x}{x+1} \quad D \in (-\infty, -\frac{1}{p}) \cup (-\frac{1}{p}, \infty)$$

$$\frac{x^p + x^p - x - 1}{x^p - 1} = \frac{(x+1)(x^p - 1)}{x^p - 1} = x+1 \quad f(x) \in \mathbb{Q}(x) \rightarrow x+1 \leq x+1 \Rightarrow x \leq x$$

$$f(1) \leq \frac{a+p}{1+b} \leq 1 \rightarrow a+p \leq 1+b \rightarrow a \leq 1+b-p \quad \left[b + (1+p) = \frac{a}{p} \right]$$

$$f(-1) = \frac{-a+p}{b-1} \leq 1 \rightarrow -a+p \leq b-1 \rightarrow -a \leq b-1-p \rightarrow a \geq 1-p-b$$

$$\frac{x+1}{x-1} = \frac{x^p-1}{x^p+am+b} \Rightarrow (x+1)(x^p+am+b) = (x-1)(x^p-1) \Rightarrow$$

$$x^p+am+b+bx+ax^p+amx+b \leq x^p+(a+1)x^p+(a+b)x+b = x^p+ax^p+ax+b = x^p+ax^p+ax+b$$

$$\Rightarrow b \leq 1 \text{ و } a+1 \leq -1 \rightarrow a \leq -2 \quad a+b \leq -1 \quad (2) \rightarrow c \leq 1-b$$

$$\frac{x+1}{x-1} \xrightarrow{x \leq 1} \frac{x^p-1}{x^p-(1+1)x+1} = \frac{(x-1)(x^{p-1}+x^{p-2}+\dots+1)}{(x-1)(x^p-1)} \xrightarrow{x \leq 1} \frac{1}{x^p-1} \quad (2) \rightarrow c \leq 1-b$$

$$f_{(n+1)} \leq ((n+1)-1)^p \times ((n+1)-1) = (n-1)^p (n) = n(n-1)^p$$

$$f_{(1-n)} \leq ((1-n)-1)^p ((1-n)-1) = (-n-1)^p (-n) \Rightarrow -n(n+1)^p$$

$$f_{(n+1)} + f_{(1-n)} \leq n(n-1)^p + -n(n+1)^p \Rightarrow n((n-1)^p - (n+1)^p) \Rightarrow$$

$$\Rightarrow n(x^p - (x-n-x^p-x)) \leq n(-1-x) = -1-x^p$$

$1 - \sqrt{1-x} = \sqrt{1-x} \rightarrow -\sqrt{1-x} + \sqrt{1-x} \rightarrow x = -1 \Rightarrow f_{(-1)} = \sqrt{1+1} = 1$ $1 - \sqrt{1-x} = -1 \Rightarrow -\sqrt{1-x} = -2 \Rightarrow \sqrt{1-x} = 2 \Rightarrow 1-x = 4 \Rightarrow x = -3 \Rightarrow f_{(-3)} = \sqrt{1+1} = 1$ $f_{(-1)} + f_{(-3)} = 1 + 1 = 2$	8
$x + \sqrt{x} = 4 \rightarrow t^2 + t - 3 = 0 \Rightarrow (t-1)(t+3) = 0 \begin{cases} t=1 \\ t=-3 \end{cases}$ $\sqrt{x} = 1 \rightarrow x = 1$ $\sqrt{x} = -3 \rightarrow x = 9$ $f(4) = f_{(1)} + f_{(9)} = 1 + 3 = 4$	9
$f_{(n-1)} = (n-1)^n + (n-1) = n^n - n^{n-1} + n - 1 + n - 1 = n^n - n^{n-1} + 2n - 2$ $f_{(1-n)} = (1-n)^n + (1-n) = 1 - n^n + n^{n-1} - n + 1 - n = 2 - n^n + n^{n-1} - 2n$ $f_{(1-n)} + f_{(n-1)} = 0$	10
$\sqrt{1-x} + 1 = 1 \Rightarrow \sqrt{1-x} = 0 \Rightarrow 1-x = 0 \Rightarrow x = 1$ $\sqrt{1-x} + 1 = -1 \Rightarrow \sqrt{1-x} = -2 \Rightarrow 1-x = 4 \Rightarrow x = -3$ $f_{(1)} = 1, f_{(-3)} = 1$ $f_{(1)} + f_{(-3)} = 2$	11
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