

$$f(n) \rightarrow \begin{matrix} n \\ -n \end{matrix}$$

$$(f(-n) - n f(n) = 2|n| + 1) - n$$

$$f(n) + n f(-n) = 2|n| + 1$$

$$f(n) = \frac{(2|n| + 1)(1 - n)}{n^2 + 1}$$

$$(n^2 + 1) f(n) = (2|n| + 1)(1 - n) \rightarrow$$

$$n \rightarrow 0 \rightarrow 2f(0) = 2 \quad f(0) = 1$$

$$f(n) + n^2 = n^2 - 2n + 2 \rightarrow f(n) = n^2 - 2n + 2$$

$$\rightarrow f(n^2) = n^4 - 2n^2 + 2$$

$$y = \frac{n^2 + n + 2}{n - 1} \rightarrow y(n - 1) = n^2 + n + 2 \Rightarrow n^2 + (1 - y)n + (y + 2) = 0$$

$$\rightarrow n = \frac{y - 1 \pm \sqrt{y^2 - 4y - 4}}{2}$$

$$R_f = (-\infty, -1] \cup [2, +\infty) \quad \rightarrow y^2 - 4y - 4 \geq 0 \quad (y - 4)(y + 1) \geq 0$$

$$\text{الف} \rightarrow y_n^2 - 2n + 1 \quad \frac{-b}{2a} = \frac{2}{1} \quad y\left(\frac{20}{14}\right) - 2\left(\frac{0}{1}\right) + 1 = \frac{20}{14} - \frac{0}{14} + \frac{1}{14} = \frac{21}{14} = \frac{3}{2} \quad \min_{\text{دا}} R = \left[-\frac{11}{14}, +\infty\right)$$

$$\rightarrow \sqrt{\frac{1}{3}n^2 + n - 8} \quad \frac{-b}{2a} = 3 \rightarrow -9 + 18 - 8 = 1 \rightarrow R = [0, 2]$$

$$\rightarrow \sqrt{n^2 - 2n^2 + 2n + 1} \rightarrow \text{بزرگترین مرتبه فرد} \rightarrow \mathbb{R} \rightarrow R = [0, +\infty)$$

$$\rightarrow \log(n^2 - 2n^2 + 2n + 1) \quad \mathbb{R}$$

$$\text{الف} \rightarrow \frac{n^2 + 1}{n - 2} \rightarrow R = \mathbb{R} - \left\{\frac{1}{2}\right\}$$

$$\rightarrow \sqrt{\frac{n^2 + 1}{n - 2}} \rightarrow \mathbb{R} - \left\{\frac{1}{2}\right\} \rightarrow [0, +\infty) - \left\{\frac{1}{2}\right\}$$

$$\rightarrow \frac{n^2 + 1}{\sqrt{n^2 + 1}} \rightarrow \frac{n^2}{0, \infty} \rightarrow R = \left[\frac{1}{\sqrt{2}}, +\infty\right) \quad \rightarrow n^2 + \frac{1}{n^2 + 1} \rightarrow \frac{n^2}{+\infty, 0} \rightarrow R = \left[\frac{1}{\sqrt{2}}, +\infty\right)$$

$$y = \frac{r n^2 + \varepsilon}{n^2 - 1} \rightarrow y n^2 - y = r n^2 + \varepsilon \rightarrow (y - r) n^2 = y + \varepsilon \quad n = \pm \sqrt{\frac{y + \varepsilon}{y - r}}$$

$$\rightarrow n^2 = 0 \rightarrow y = -\varepsilon$$

$$n^2 = \infty \rightarrow y = r \rightarrow \underline{\underline{y = r}}$$

$$R = (-\infty, -\varepsilon] \cup (r, +\infty)$$

على

$$+ \frac{-\varepsilon}{1} - \frac{r}{1} +$$

$$R_f = (-\infty, -\varepsilon] \cup (r, +\infty)$$

تذكر

$$f(n) = \frac{r \sin n + \varepsilon}{\sin n - r}$$

$$\rightarrow \sin n_{\max} = 1 \rightarrow \frac{r + \varepsilon}{1 - r} = \underline{\underline{-\frac{r}{1-r}}}$$

$$\sin n_{\min} = -1 \rightarrow \frac{-r + \varepsilon}{-1 - r} = \underline{\underline{-\frac{1}{r}}}$$

مخرج مخالف
صفر

$$R_f = [-\frac{r}{1-r}, -\frac{1}{r}]$$

↓
دخل

$$f \rightarrow y = \left| \frac{r n - 1}{\varepsilon n} \right| \rightarrow \frac{r n - 1}{\varepsilon n} \sim \mathbb{R} - \left\{ \frac{r}{\varepsilon} \right\} \xrightarrow{\text{قدر مطلق}} \left| -\frac{r}{\varepsilon} \right| = \left| \frac{r}{\varepsilon} \right|$$

$$| -\infty | = | \infty |$$

$$\Rightarrow R_{\text{الف}} = [0, +\infty)$$

$$f \rightarrow y = \sqrt{\frac{9n - 18}{-18 + \varepsilon n}} \sim \sqrt{\frac{9(n - 2)}{\varepsilon(n - 2)}} \xrightarrow{n \neq 2} \sqrt{\frac{9}{\varepsilon}} = \frac{r}{\varepsilon} \quad R_f = \left\{ \frac{r}{\varepsilon} \right\}$$

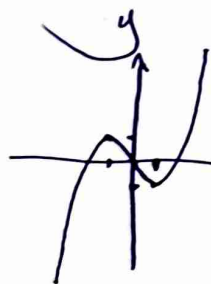
$$y = x |x| - r x$$

$$(n > 0) \rightarrow n^2 - r n$$

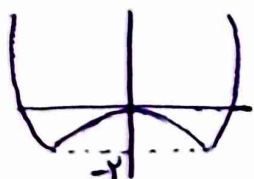
$$\text{ex: } (1, -1)$$

$$(n < 0) \rightarrow -n^2 - r n$$

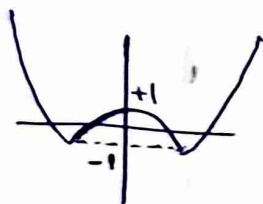
$$\text{ex: } (-1, 1)$$



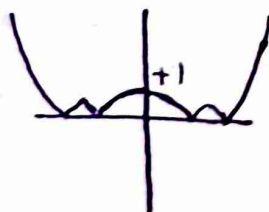
$$R_f = \mathbb{R}$$



$$y = f(|x|)$$



$$y = f(|x|) + 1$$



$$y = |f(|x|) + 1|$$