

$$f(-n) = 2f(n) = 2/|n|+1 \xrightarrow{n \rightarrow -n} f(-(-n)) = (-n)f(-n) = 2/|-n|+1 \xrightarrow{|-n|=|n|} f(n) + n f(-n) = 2/|n|+1$$

$$\Downarrow \textcircled{1}$$

$$f(-n) = 2f(n) + 2/|n|+1 \textcircled{2}$$

$$\xrightarrow{\textcircled{1}} f(n) = \frac{(1-n)(2/|n|+1)}{n^2+1}$$

$$f(n) = \frac{2/|n|(1-n) + (1-n)}{n^2+1} \leftarrow f(n)(n^2+1) = 2/|n|+1 - 2n/|n| \cdot n \leftarrow 2^2 f(n) + 2n/|n| + n + f(n) = 2/|n|+1$$

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$$f(x) + \frac{f(x)}{x} = x^2 - 2x + 3 \xrightarrow{x \rightarrow -x} x f(x) + (-x) f(x) = 4 \Rightarrow f(x) = x^2 - 2x + 3$$

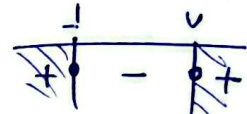
$$\xrightarrow{x \rightarrow -x} f(x^2) = (x^2)^2 - 2(x^2) + 3 = x^4 - 2x^2 + 3$$

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$$y = \frac{x^2 + n + 2}{n-1} \rightarrow y(n-y) = x^2 + n + 2 \Rightarrow x^2 + (1-y)n + (y+2) = 0 \Rightarrow \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(1-y) \pm \sqrt{(1-y)^2 - 4(1)(y+2)}}{2}$$

$$\Rightarrow \Delta \geq 0 \Rightarrow (1-y)^2 - 4(1)(y+2) \geq 0 \Rightarrow y^2 - 6y - 7 \geq 0 \Rightarrow (y-7)(y+1) \geq 0$$

$$\Rightarrow R_f = (-\infty, -1] \cup [7, +\infty)$$



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$$1) y = \frac{y^2 - 2n + 1}{n-1} \Rightarrow \left| \frac{\frac{y^2}{n} - 2}{-1/\frac{1}{n}} \right| \Rightarrow R_f = \left[-\frac{1}{\sqrt{n}}, +\infty \right)$$

$$2) y = \log(n^2 - 4n^2 + 4n + 2) \Rightarrow \log(1R) = \log(-\infty, +\infty) \Rightarrow \log(0, +\infty) \Rightarrow R_f = (-\infty, +\infty)$$

$$\Rightarrow y = \sqrt{\frac{y^2 + 4n - 2}{n-1}} \Rightarrow \left| \frac{\frac{y^2}{n} - 2}{-1/\frac{1}{n}} \right| \Rightarrow \sqrt{\frac{y^2 + 4n - 2}{n-1}} \Rightarrow [0, 2] : R_f$$

$$2) y = \sqrt{n^2 - 2n^2 + 4n + 1} \Rightarrow \sqrt{R} \Rightarrow R_f = [0, +\infty)$$

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$$1) y = \frac{y^2 + 1}{y - 4} \Rightarrow 1R - \{4\} \Rightarrow 1R - \left\{ \frac{4}{y} \right\} : R_f$$

$$\Rightarrow y = \sqrt{\frac{y^2 + 1}{y - 4}} \Rightarrow \sqrt{1R - \{4\}} \Rightarrow R_f = [0, +\infty) - \{4\}$$

$$2) y = \frac{y^2 + 1}{\sqrt{y^2 + 3}} = \frac{y^2 + 3}{\sqrt{y^2 + 3}} + \frac{1}{\sqrt{y^2 + 3}} = \frac{1}{\sqrt{y^2 + 3}} + \sqrt{\frac{y^2 + 3}{y^2 + 3}}$$

$$\xrightarrow{\min \sqrt{y^2 + 3}} R_f = \left[\frac{1}{\sqrt{3}}, +\infty \right) = \left[\frac{\sqrt{3}}{3}, +\infty \right)$$

$$3) y = n^2 + \frac{1}{n^2 + 4} \xrightarrow{x^2} (n^2 + 4) \left(\frac{1}{n^2 + 4} \right) - 4 \xrightarrow{\min(n^2 + 4) = 4} R_f = \left[\frac{1}{4}, +\infty \right)$$

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$y = \frac{x^2 + x}{x^2 - 1} \Rightarrow y = \frac{x^2 + x}{x^2 - 1} \Rightarrow (x-y)x^2 + (y+x) = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow \Delta \geq 0$ $(1) y + x + 1 = 0 \Rightarrow x = -y - 1$ $(y-1)(y+1) \leq y^2 + y - 1 \leq 0 \Rightarrow (0)^2 - 1 \leq (y+1)^2$ $R_f: (-\infty, -1] \cup (1, +\infty)$ $y = \frac{x^2 + x}{x^2 - 1} \xrightarrow{x \rightarrow \pm\infty} \frac{x^2}{x^2} = 1$ $\xrightarrow{x \rightarrow -1} \frac{1-1}{1-1} = \frac{0}{0}$ $\xrightarrow{x \rightarrow 1} \frac{1+1}{1-1} = \frac{2}{0}$ $R_f: (-\infty, -1] \cup (1, +\infty)$	6
$f(x) = \frac{x \sin x + x}{\sin x - 1} \xrightarrow{\sin x = 1} \frac{x + x}{1 - 1} = \frac{2x}{0} = \pm\infty$ $\xrightarrow{\sin x = -1} \frac{-x + x}{-1 - 1} = \frac{0}{-2} = 0$ $R_f: [-\infty, 0] \cup (0, +\infty)$ $-1/2 \sin x \leq 1$	7
$y = \left \frac{x-1}{x+1} \right \Rightarrow R - \{ \frac{1}{2} \} \Rightarrow R_f: [0, +\infty) - \frac{1}{2}$ $\Rightarrow y = \sqrt{\frac{9x-1}{x-1}} \Rightarrow \sqrt{R - \{ \frac{1}{9} \}} \Rightarrow [0, +\infty) - \frac{1}{9}$	8
$y = x x - x_n \rightarrow \begin{cases} x \geq 0; y = x(x) - x_n = x^2 - x_n \Rightarrow y = x^2 - x_n \Rightarrow \frac{b}{4a} = 1 \Rightarrow R_1: [-1, +\infty) \text{ (I)} \\ x < 0; y = x(-x) - x_n = -x^2 - x_n \Rightarrow \frac{b}{4a} = -1 \Rightarrow R_2: (-\infty, 1] \text{ (II)} \end{cases}$ $R_f: \text{(I)} \cup \text{(II)}: (-\infty, 1] \cup [-1, +\infty) \Rightarrow R_f = R$	9
$f(x) = \frac{x}{x^2 + 1} \Rightarrow f(x) = \frac{ x }{x^2 + 1} \Rightarrow f(x) + 1 = \frac{ x }{x^2 + 1} + 1$ $f(x) + 1 \Rightarrow y = f(x) + 1$	10