

$$f(-n) - n f(n) = 2|n| + 1 \rightarrow \textcircled{A} f(-n) - n^2 f(n) = 2|n|n + n$$

$$\textcircled{B} f(n) + n f(-n) = 2|n| + 1 \xrightarrow{\textcircled{A}-\textcircled{B}} n^2 f(n) + f(n) = -2|n|n - n$$

$$f(n) = \frac{-2|n|n - n + 2|n| + 1}{n^2 + 1}$$

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$$f(n) + f(0) = n^2 - 2n + 6$$

$$f(0) + f(0) = 6$$

$$f(0) = 3$$

$$f(n) = n^2 - 2n + 3$$

$$f(n^2) = n^4 - 2n^2 + 3$$

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$$g_n - y = n^2 + 2n + 2 \quad n^2 + (1-y)n + 2 + y = 0$$

$$n = \frac{y-1 \pm \sqrt{y^2 - 2y + 1 - 1 - 4y}}{2}$$

$$y^2 - 2y - 1 \geq 0$$

$$(y-1)(y+1)$$

$$+ \frac{-1}{2} - \frac{1}{2} +$$

$$y \in [1, \infty) \cup (-\infty, -1]$$

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$$\text{الف) } y = 2n^2 - 8n + 1 \quad \frac{-b}{2a} = \frac{-8}{4} = -2$$

$$\frac{2(\frac{16}{4})}{\frac{16}{4}} + \frac{2 \cdot 8}{\frac{16}{4}} + \frac{1}{1} = \frac{13}{1}$$

$$R: [\frac{13}{4}, +\infty)$$

$$\text{ب) } y = \sqrt{-n^2 + 4n - 5}$$

$$\frac{-4}{-2} = 2 \quad -\frac{9+1-8}{4} = \frac{0}{4} = 0 \quad \text{min}$$

$$R: [0, 2]$$

$$\text{ج) } y = \sqrt{n^2 - 2n^2 + 4n + 1} \Rightarrow R: [0, +\infty)$$

$$\text{د) } y = \log(n^2 - 2n^2 + 4n + 1) \quad R: (-\infty, +\infty)$$

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$$\text{الف) } y = \frac{2n+1}{n^2-2} \quad n = \frac{2y+1}{y^2-2} \quad y \neq \frac{1}{\sqrt{2}}$$

$$\text{ب) } y = \frac{2n-5}{\sqrt{4n+1}} \quad y^2 = \frac{4n+1}{n-2} \quad n = \frac{2y^2-1}{y^2+2} \quad y \in [0, +\infty)$$

$$\text{ج) } y = \frac{n^2+3+1}{\sqrt{n^2+2}} = \sqrt{n^2+2} + \frac{1}{\sqrt{n^2+2}} \quad \sqrt{n^2+2} \neq 1 \quad \text{min} = \sqrt{2}$$

$$y \geq \sqrt{2} + \frac{1}{\sqrt{2}} = \frac{3}{\sqrt{2}}$$

$$\text{د) } y = 2n + \frac{1}{2n+2} \quad y+2 \geq 2 + \frac{1}{2} \quad y \geq \frac{5}{2}$$

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$$y = \frac{x^2 + 5}{x - 2} \quad x \neq 2 \quad y \in (-\infty, -1] \cup (7, +\infty)$$

$$y \cdot (x - 2) = x^2 + 5 \quad -x^2 + 2y = x^2 + 5 \Rightarrow 2y = \frac{x^2 + 5}{x - 2}$$

$$\sin x = \frac{f(x) + 5}{f(x) - 2} \quad -1 \leq \frac{f(x) + 5}{f(x) - 2} \leq 1 \quad R: [-2, \frac{1}{2}]$$

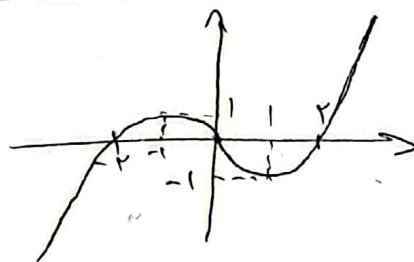
$$\frac{f(x) + 5}{f(x) - 2} \leq 1 \quad \frac{-5}{f(x) - 2} \leq 0 \quad \frac{-5}{f(x) - 2} \leq 0 \quad [-2, 2) \quad \wedge$$

$$\frac{f(x) + 5}{f(x) - 2} \geq -1 \quad \frac{f(x) + 5}{f(x) - 2} \geq -1 \quad \frac{f(x) + 5}{f(x) - 2} \geq -1 \quad (-\infty, \frac{1}{2}] \cup (2, +\infty)$$

$$\lim_{n \rightarrow \infty} \left| \frac{x_n - 1}{x_n} \right| = 0 \quad R: [0, +\infty)$$

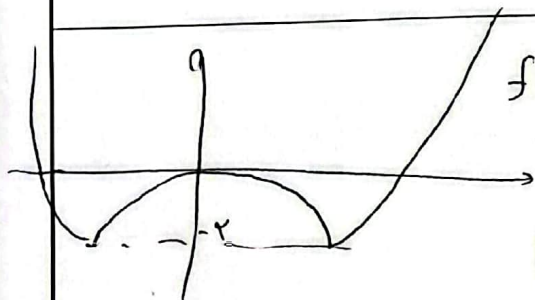
$$y = \sqrt{\frac{x^2 - 1}{x^2 + 1}} = \sqrt{\frac{x^2 - 1}{x^2 + 1}} = \frac{x}{x + 1} \quad R: \left\{ \frac{x}{x + 1} \right\}$$

$$y = \begin{cases} x^2 - 2x & x \geq 0 \\ -x^2 - 2x & x \leq 0 \end{cases}$$

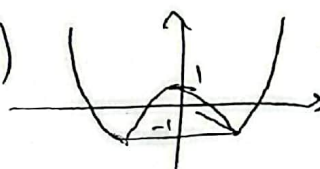


$$R: (-\infty, +\infty)$$

$$R \cup \omega$$



$$f(x)$$



$$f(x) + 1$$



$$|f(x) + 1|$$