

$$f(-n) - nf(n) = c|n| + 1 \xrightarrow{n=-n} f(-(-n)) - (-n)f(-n) = f(n) + nf(-n) = c|n| + 1 \quad (1)$$

$$\begin{aligned} f(-n) - nf(n) = c|n| + 1 &\Rightarrow f(-n) = c|n| + 1 + nf(n) \xrightarrow{\text{جایگزینی در رابطه (1)}} f(n) + n(c|n| + 1 + nf(n)) = c|n| + 1 \\ \Rightarrow f(n) + n|n| + n + n^2 f(n) &= c|n| + 1 \Rightarrow f(n) + n^2 f(n) + n|n| + n = c|n| + 1 \Rightarrow f(n)(1 + n^2) + n(|n| + 1) = c|n| + 1 \\ \Rightarrow f(n)(1 + n^2) &= c|n| + 1 - n(|n| + 1) \Rightarrow f(n)(n^2 + 1) = (c|n| + 1)(1 - n) \Rightarrow f(n) = \frac{(c|n| + 1)(1 - n)}{n^2 + 1} \end{aligned}$$

$$\begin{aligned} f(n) + f(1) = n^2 - n + 4 &\xrightarrow{n=1} f(1) = 1^2 - 1 + 4 = 4 \Rightarrow f(1) = 4 \Rightarrow f(n) = n^2 - n + 4 \\ \Rightarrow f(n^2) &= (n^2)^2 - n^2 + 4 = n^4 - n^2 + 4 \\ \Rightarrow f(n^2) &= n^4 - n^2 + 4 \end{aligned} \quad (2)$$

$$\begin{aligned} y = \frac{n^2 + n + 1}{n - 1} &\Rightarrow y(n - 1) = n^2 + n + 1 \Rightarrow n^2 - y(n - 1) + n + y + 1 = 0 \Rightarrow n^2 + n(1 - y) + (y + 1) = 0 \\ (1 - y)^2 - 4(1)(y + 1) &\geq 0 \Rightarrow y^2 - 2y + 1 - 4y - 4 \geq 0 \Rightarrow y^2 - 6y - 3 \geq 0 \\ \Rightarrow y &\in (-\infty, -1] \cup [4, +\infty) \\ \Rightarrow R_f &= (-\infty, -1] \cup [4, +\infty) \end{aligned} \quad (3)$$

$$\text{الف) } y = c(n^2 - 2n + 1) \xrightarrow{\text{در } n=1 \text{ به } \min} n \geq \frac{b}{a} = \frac{-(-2)}{c} = \frac{2}{c} \Rightarrow y \geq \frac{-10}{1} \Rightarrow R_f = \left[\frac{-10}{1}, +\infty \right)$$

$$\begin{aligned} \text{ب) } y &\geq \sqrt{n^2 + 4n - 4} \Rightarrow f(n) = n^2 + 4n - 4 \Rightarrow n \geq \frac{-4}{-2} = 2 \Rightarrow y \geq 4 \Rightarrow R_f = [4, +\infty) \\ \Rightarrow \sqrt{f(n)} &= R_f = [4, +\infty) \end{aligned}$$

$$\text{ج) } y = \sqrt{n^2 - 4n^2 + 5n + 1} \Rightarrow f(n) = n^2 - 4n^2 + 5n + 1 \Rightarrow R_f = (-\infty, +\infty) \Rightarrow \sqrt{f(n)} = R_f = [0, +\infty)$$

$$\begin{aligned} \text{د) } \log(n^2 - 4n^2 + 5n + 1) &\Rightarrow n^2 - 4n^2 + 5n + 1 > 0 \Rightarrow \min f(n) = 0^+ \quad \left. \begin{array}{l} \log(0^+) = -\infty \\ \log(+\infty) = +\infty \end{array} \right\} \\ \Rightarrow R_f &= (-\infty, +\infty) \subseteq \mathbb{R} \end{aligned}$$

$$\text{الف) } y = \frac{(n+1)}{n-1} \quad R_f = \mathbb{R} - \left\{ \frac{1}{a} \right\} \Rightarrow R_f = \mathbb{R} - \left\{ \frac{1}{c} \right\}$$

$$\text{ب) } y = \sqrt{\frac{(n+1)}{n-1}} \Rightarrow f(n) = \frac{(n+1)}{n-1} \Rightarrow R_f = [1, +\infty) - \left\{ \frac{1}{c} \right\} \Rightarrow R_f = [1, +\infty) - \left\{ \frac{1}{c} \right\}$$

برای نیلوسان کلاس دوازدهم سپر B تکلیف سری ۴

$$ج) y = \frac{x^2 + 4}{\sqrt{x^2 + 4}} \xrightarrow[\text{همواره مثبت}]{\text{صورت و مخرج}} \text{Min}(u) = \dots \xrightarrow{u=2} \frac{4}{\sqrt{4}} = \frac{4}{2} = 2 \Rightarrow R_f: [\frac{4}{2}, +\infty)$$

$$و) y = x^2 + \frac{1}{x^2 + 4} \xrightarrow[\text{افزودن و کاستن}]{a} (x^2 + 4) + \frac{1}{(x^2 + 4)} - 4 \Rightarrow y = a + \frac{1}{a} - 4, \text{Min}(a) = 4$$

$$\Rightarrow \text{Min}: 4 + \frac{1}{4} - 4 = \frac{1}{4} \Rightarrow R_f: [\frac{1}{4}, +\infty)$$

۶) $y = \frac{3x^2 + 4}{x^2 - 1}$ روش اول: $y x^2 - y = 3x^2 + 4 \Rightarrow x^2 - y x^2 + y + 4 = 0 \Rightarrow x^2(1 - y) + y + 4 = 0 \Rightarrow x^2 = \frac{-y - 4}{1 - y}$

$$\Rightarrow x^2 = \frac{y + 4}{y - 1} \Rightarrow x = \sqrt{\frac{y + 4}{y - 1}} \Rightarrow \frac{y + 4}{y - 1} \geq 0, y \neq 1 \Rightarrow R_f: (-\infty, -4] \cup (1, +\infty)$$

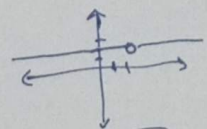
روش دوم: $\text{Min}(x^2) = 0 \Rightarrow \frac{0 + 4}{0 - 1} = -4$
 $\text{Max}(x^2) = +\infty \Rightarrow \frac{+\infty + 4}{+\infty - 1} \approx +1$

چون نمی‌توانیم از $\frac{1}{0}$ استفاده کنیم

$$R_f: (-\infty, -4] \cup (1, +\infty)$$

۷) $f(x) = \frac{2 \sin x + 4}{\sin x - 2}$ $\left. \begin{array}{l} \text{Max}(\sin x) = 1 \\ \text{Min}(\sin x) = -1 \end{array} \right\} \Rightarrow \frac{2 + 4}{1 - 2} = -6, \frac{-2 + 4}{-1 - 2} = \frac{2}{-3} = -\frac{2}{3} \Rightarrow R_f: [-6, -\frac{2}{3}]$

۸) الف) $y = \left| \frac{3x - 1}{5x} \right| \xrightarrow[\text{همواره مثبت}]{\text{قدر مطلق}} R_f: [0, +\infty)$



ب) $y = \sqrt{\frac{4x - 1}{-x + 5}}$ $\rightarrow \frac{4x - 1}{5x - 1} = \frac{4(x - 1)}{5(x - 1)} \xrightarrow{x \neq 1} \frac{4}{5} \Rightarrow y = \sqrt{\frac{4}{5}} = \frac{2}{\sqrt{5}} \Rightarrow R_f: \left\{ \frac{2}{\sqrt{5}} \right\}$

۹) $y = x|x| - 5x$

اگر $x > 0$: $x^2 - 5x = x^2 - 5x + 1 - 1 = (x - 1)^2 - 1$

اگر $x < 0$: $-x^2 - 5x = -(\frac{1}{4}x^2 + \frac{5}{2}x + \frac{25}{4}) + \frac{25}{4} + 1 - 1 = -(\frac{1}{4}x + \frac{5}{2})^2 + \frac{25}{4} + 1 - 1 = -(\frac{1}{4}x + \frac{5}{2})^2 + \frac{27}{4}$

نتیجه: $R_f: \mathbb{R}$

۱۰) تابع $f(x)$ نسبت به محور y متقارن است، از اینجایی $f(x) = f(|x|)$

