

$$f(-x) - x f(x) = 2|x| + 1 \Rightarrow x - x \Rightarrow -x f(-x) + x^2 f(x) = -2|x| - x$$

$$f(x) + x(f(-x)) = 2|x| + 1$$

$$f(x) + x f(-x) = 2|x| + 1$$

$$f(x)(x^2 + 1) = 2|x| + 1 - 2|x| - x$$

$$f(x) = \frac{-2x|x| + 2|x| - x + 1}{x^2 + 1}$$

$$f(x) = \frac{(2|x| + 1)(1 - x)}{x^2 + 1}$$

$$f(x) + f(0) = x^2 - 3x + 3$$

$$x=0 \Rightarrow 2f(0) = 3$$

$$f(0) = 3$$

$$f(x) + 3 = x^2 - 3x + 3 \xrightarrow{x \leq t} f(t) = t^2 - 3t + 3$$

$$f(x) = x^2 - 3x + 3$$

$$f(x^2) = x^4 - 3x^2 + 3$$

$$y = \frac{x^2 + x + 2}{x - 1}$$

$$yx - y = x^2 + x + 2$$

$$x^2 + x - yx + y + 2 = 0$$

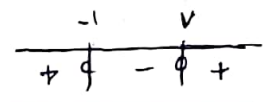
$$x^2 + (1 - y)x + y + 2 = 0$$

$$\Delta \geq 0$$

$$y^2 - 4y + 1 - 4(y + 2) \geq 0$$

$$y^2 - 4y - 7 \geq 0$$

$$(y + 1)(y - 7) \geq 0$$



$$Rf: (-\infty, -1] \cup [7, +\infty)$$

الف) $x_{\min} = \frac{-b}{2a} = \frac{d}{f} \Rightarrow y_{\min} = \frac{-14}{1} \Rightarrow Rf: [-\frac{14}{1}, +\infty)$

ب) $x_{\max} = \frac{-b}{2a} = 3 \Rightarrow y_{\max} = 4 \Rightarrow Rf: [0, 4]$

ج) $\sqrt{x} \Rightarrow Rf: [0, +\infty)$

د) $\log x \Rightarrow Rf: (-\infty, +\infty) \subseteq \mathbb{R}$

الف) $Rf: \mathbb{R} - \{\frac{a}{c}\} \Rightarrow Rf: \mathbb{R} - \{\frac{3}{4}\}$

ب) $Rf: \mathbb{R} - \{\frac{a}{c}\} \Rightarrow \mathbb{R} - \{4\} \Rightarrow Rf: [0, +\infty) - \{4\}$

ج) $\frac{x^2 + 3}{\sqrt{x^2 + 3}} + \frac{1}{\sqrt{x^2 + 3}} \Rightarrow a + \frac{1}{a} \geq 2$
 $a + \frac{1}{a} \leq -2$
 $x^2 > 0$ خارج نمی‌تواند و $\min$ می‌گیریم $\xrightarrow{x=0} \frac{3}{\sqrt{3}} + \frac{1}{\sqrt{3}} = \frac{4}{\sqrt{3}}$
 $Rf: [\frac{4}{\sqrt{3}}, +\infty)$

د) $x^2 + 4 + \frac{1}{x^2 + 4} - 4 \Rightarrow a + \frac{1}{a} \geq 2$
 $a + \frac{1}{a} \leq -2$
 $x^2 > 0$ خارج نمی‌تواند و $\min$ می‌گیریم $\xrightarrow{x=0} 4 + \frac{1}{4} = \frac{17}{4}$
 $Rf: [\frac{17}{4}, +\infty)$

$$y x^2 - y = 3x + 4 \leadsto x^2(y-3) = 4+y \quad x^2 \geq 0 \quad \frac{-4}{y-3} \leq x^2 \leq \frac{4+y}{y-3}$$

$$-3x^2 + yx^2 = 4+y \quad x^2 = \frac{y+4}{y-3} \leadsto \frac{y+4}{y-3} \geq 0 \quad \frac{-4}{y-3} \leq x^2 \leq \frac{4+y}{y-3}$$

$$Rf: (-\infty, -4] \cup (3, +\infty)$$

$$x^2 = 0 \leadsto y = -4 \quad \text{خرج می تواند باشد} \quad Rf: (-\infty, -4] \cup (3, +\infty)$$

$$x^2 = +\infty \leadsto y = 3$$

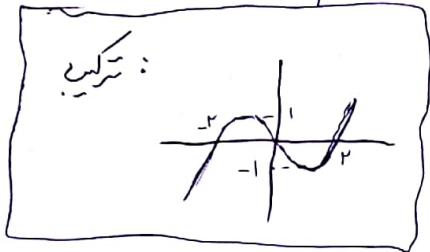
$$\sin \max = 1 \leadsto y = -3 \quad \text{خرج می تواند باشد} \quad Rf: [-3, -\frac{1}{3}]$$

$$\sin \min = -1 \leadsto y = -\frac{1}{3}$$

$$\text{الف} \quad R - \left\{ \frac{a}{c} \right\} \leadsto R - \left\{ \frac{3}{4} \right\} \Rightarrow |R - \left\{ \frac{3}{4} \right\}| \quad Rf: [0, +\infty) - \left\{ \frac{3}{4} \right\}$$

$$\text{ب) } \sqrt{\frac{9x-18}{4x-8}} = \sqrt{\frac{9(x-2)}{4(x-2)}} = \sqrt{\frac{9}{4}} = \left| \frac{3}{2} \right| \leadsto Rf: \frac{3}{2}$$

$$x \geq 0 \leadsto y = x^2 - 2x \quad x(x-2)$$

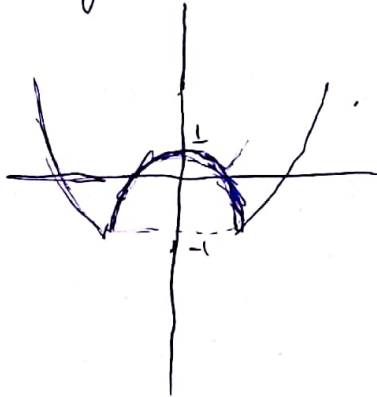



$$x < 0 \leadsto y = -x^2 - 2x \quad -x(x+2)$$



اگر $f(x) = f(x)$ فنحن حذف نموداری
قرینه نمودار باز همان می شود

$$y = f(x) + 1$$



$$y = |f(x) + 1|$$

