

$$\begin{aligned} f(-n) - n f(n) &= 2|n| + 1 \rightarrow n f(-n) - n^2 f(n) = 2n|n| + n \\ f(n) + n f(-n) &= 2|-n| + 1 \\ \rightarrow f(n)(1+n^2) &= \frac{2|-n| - n - 2n|n| + 1}{n^2 + 1} \rightarrow f(n) = \begin{cases} \frac{2n - n - 2n^2 + 1}{n^2 + 1} = \frac{-2n^2 + n + 1}{n^2 + 1} & n > 0 \\ \frac{-2n - n + 2n^2 + 1}{n^2 + 1} = \frac{2n^2 - 3n + 1}{n^2 + 1} & n < 0 \end{cases} \end{aligned}$$

$$\begin{aligned} f(n) + f(-1) &= n^2 - 3n + 3 \xrightarrow{n=0} f(0) = 3 \rightarrow f(0) = 3 \\ \rightarrow f(n) &= n^2 - 3n + 3 \rightarrow f(n^2) = n^4 - 3n^2 + 3 \end{aligned}$$

$$\begin{aligned} y &= \frac{n^2 + n + 1}{n - 1} \rightarrow yn - y = n^2 + n + 1 \rightarrow n^2 + (1-y)n + (y+1) = 0 \\ \Delta \geq 0 &\rightarrow (1-y)^2 - 4(y+1) \geq 0 \rightarrow y^2 - 4y - 3 \geq 0 \rightarrow (y-7)(y+1) \geq 0 \\ \rightarrow R &= (-\infty, -1] \cup [7, +\infty) \end{aligned}$$

$$\begin{aligned} \text{الف)} \quad y &= 2n^2 - 3n + 1 \cup \quad ns = \frac{a}{p} \rightarrow ys = \frac{2a}{18} - \frac{3a}{9} + 1 \rightarrow ys = 1 - \frac{2a}{9} = \frac{14}{9} \\ \rightarrow R &= \left[\frac{14}{9}, +\infty \right) \end{aligned}$$

$$\begin{aligned} \text{ب)} \quad y &= \sqrt{-n^2 + 4n - 5} \rightarrow -n^2 + 4n - 5 \geq 0 \Rightarrow ns = \frac{-4}{-2} = 2 \Rightarrow ys = -9 + 18 - 5 \\ &= 4 \\ [-\infty, 5] &\rightarrow R = [0, 4] \end{aligned}$$

$$\text{ج)} \Rightarrow y = \sqrt{2n^2 - 3n + 1} \rightarrow (-\infty, +\infty) \rightarrow R = [0, +\infty)$$

$$\text{د)} \Rightarrow \log(2n^2 - 4n^2 + 4n + 1) \rightarrow n^2 - 2n^2 + 4n + 1 \rightarrow (0, +\infty) \Rightarrow R = (-\infty, +\infty)$$

$$\text{الف)} \quad y = \frac{3n+1}{2n-5} \rightarrow R = \mathbb{R} - \left\{ \frac{3}{2} \right\}$$

$$\text{ب)} \quad y = \sqrt{\frac{5n+1}{n-2}} \rightarrow \mathbb{R} - \left\{ \frac{1}{2} \right\} \rightarrow R = (0, +\infty) - \left\{ \frac{1}{2} \right\}$$

$$\text{ج)} \quad y = \frac{n^2 + 4}{\sqrt{n^2 + 3}} = \sqrt{n^2 + 3} + \frac{1}{\sqrt{n^2 + 3}} \rightarrow \frac{1}{\sqrt{3}} + \sqrt{3} = \frac{4\sqrt{3}}{3} \rightarrow \left[\frac{4\sqrt{3}}{3}, +\infty \right) = \mathbb{R}$$

$$\text{د)} \quad n^2 + \frac{1}{2n+5} = y \rightarrow y = n^2 + 5 + \frac{1}{2n+5} - 5 \rightarrow y = 5 + \frac{1}{2n+5} - 5 = \frac{1}{2n+5} \rightarrow R = \left[\frac{1}{2}, +\infty \right)$$

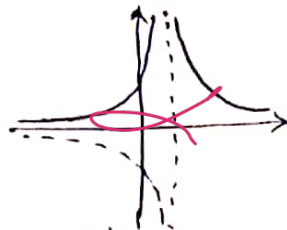
$$y = \frac{2n^2 + 4}{n^2 - 1} \rightarrow \text{روش اول} \rightarrow yn^2 - y - 2n^2 - 4 = 0 \rightarrow (y-2)n^2 + (-4-y) = 0 \rightarrow (y-2)n^2 + (-4-y) = 0$$

$$\Delta \geq 0 \Rightarrow 0 + 4(4+y)(y-2) \geq 0 \Rightarrow (4+y)(y-2) \geq 0 \rightarrow \frac{-4}{1} \quad \frac{2}{1} \rightarrow R = (-\infty, -4] \cup [2, +\infty)$$

$$\text{روش دوم} \quad n^2 \rightarrow [0, +\infty) \rightarrow [-4, 2] \rightarrow \text{برای استفاده از روش دلتا، ضریب } n^2 \text{ باید منفی شود! ابتدا اصلاً امتحان به استفاده از این روش هم کنید.}$$

$$\begin{aligned} f(x) &= \frac{2 \sin x + 4}{\sin x - 2} \rightarrow -1 \leq \sin x \leq 1 \\ &\rightarrow \left[-\frac{1}{2}, \frac{1}{2} \right] = \mathbb{R} \end{aligned}$$

$$الف) y = \left| \frac{x-1}{x} \right|$$



$$R = (0, +\infty)$$

٥ - ١

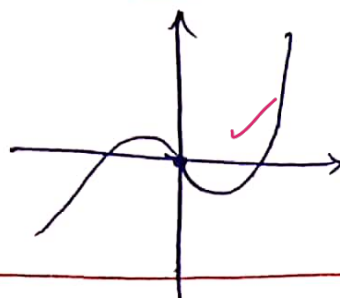
$$ب) \sqrt{\frac{9x-18}{x-1}}$$

تابع ثابت



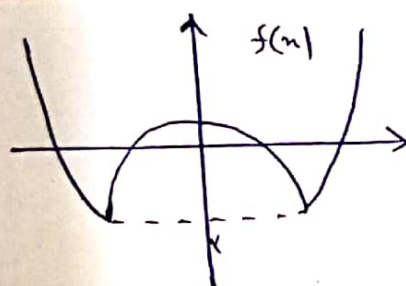
$$R = (0, +\infty) \setminus \left\{ \frac{1}{3} \right\}$$

$$y = \begin{cases} x \geq 0 \rightarrow x^2 - 2x \\ x < 0 \rightarrow -x^2 - 2x \end{cases}$$

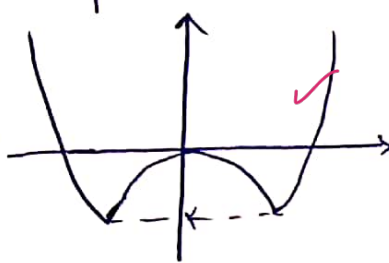


$$R = \mathbb{R}$$

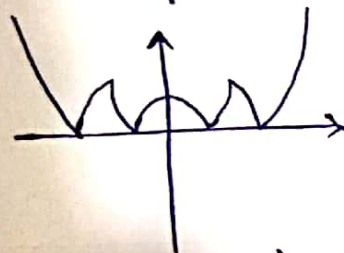
٢ - ٩



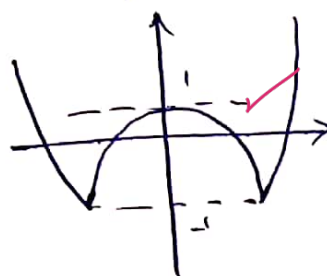
$$f(|x|)$$



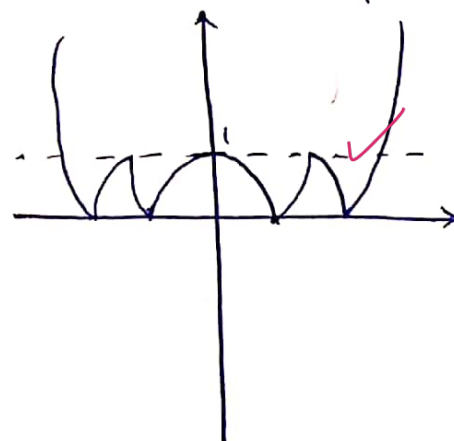
$$f(|x|) + 1$$



$$y = |f(|x|) + 1|$$



شعب



$$R = [0, +\infty)$$

٣ - ٦

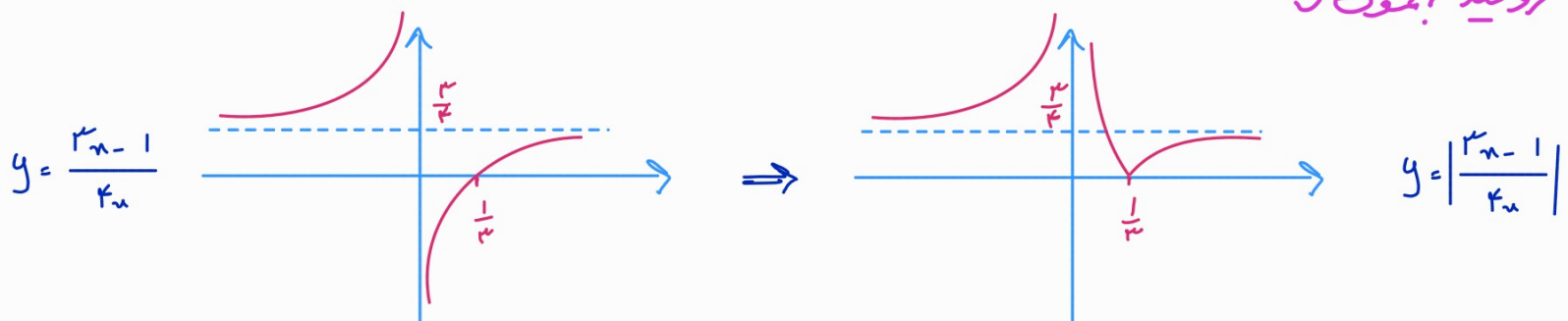
۶- روش اول: $y \cdot x^2 - y = 3x^2 + 4 \rightarrow (y-3)x^2 = y+4$

$x^2 = \frac{y+4}{y-3} \geq 0 \rightarrow \frac{-4}{+} \frac{3}{-} \Rightarrow R_y = (-\infty, -4] \cup (3, +\infty)$

نکته: $\square = x^2 \rightarrow \begin{cases} x^2 = 0 \rightarrow y = -4 \\ x^2 \sim \infty \rightarrow y = 3 \end{cases} \xrightarrow[\text{محدوده}]{\text{محدوده میوه}} R_y = (-\infty, -4] \cup (3, +\infty)$

۷- $\square = \sin x \rightarrow \begin{cases} \sin x = -1 \rightarrow y = -\frac{1}{2} \\ \sin x = 1 \rightarrow y = -3 \end{cases} \xrightarrow[\text{محدوده}]{\text{محدوده میوه}} R_y = [-3, -\frac{1}{2}]$

۸- اول برد عبارت داخل قدر مطلق
رد حساب می‌کنیم بعد قدر مطلق را بگیریم
فقط حواست باشد که $y = \frac{3}{4}$ هم هست داخل برد
 $y = -\frac{3}{4}$ عضو برد عبارت داخل قدر مطلق هست و وقتی قدر مطلق بگیریم $y = \frac{3}{4}$
روید به همون نت



۸- ب) $y = \sqrt{\frac{4(x-2)}{x(x-2)}} = \sqrt{\frac{4}{x}} = \frac{2}{\sqrt{x}} \rightarrow R_y = \{\frac{2}{\sqrt{x}}\}$