

$$\begin{aligned} f(-n) - n f(n) &= 2|n| + 1 \rightarrow n f(-n) - n^2 f(n) = 2n|n| + n \\ f(n) + n f(-n) &= 2|n| + 1 \\ \rightarrow f(n)(1+n^2) &= \frac{2|n| + 1}{n^2 + 1} \rightarrow f(n) = \begin{cases} \frac{2n - n - 2n^2 + 1}{n^2 + 1} = \frac{-2n^2 + n + 1}{n^2 + 1} & n > 0 \\ \frac{-2n - n + 2n^2 + 1}{n^2 + 1} = \frac{2n^2 - 3n + 1}{n^2 + 1} & n < 0 \end{cases} \end{aligned}$$

$$\begin{aligned} f(n) + f(-n) &= 2n^2 - 3n + 1 \xrightarrow{n=0} 2f(0) = 1 \rightarrow f(0) = \frac{1}{2} \\ \rightarrow f(n) &= n^2 - 3n + 1 \rightarrow f(n^2) = n^4 - 3n^2 + 1 \end{aligned}$$

$$\begin{aligned} y &= \frac{n^2 + n + 1}{n-1} \rightarrow yn - y = n^2 + n + 1 \rightarrow n^2 + (1-y)n + (y+1) = 0 \\ \Delta \geq 0 &\rightarrow (1-y)^2 - 4(y+1) \geq 0 \rightarrow y^2 - 4y - 3 \geq 0 \rightarrow (y-7)(y+1) \geq 0 \\ \rightarrow R &= (-\infty, -1] \cup [7, +\infty) \end{aligned}$$

$$\begin{aligned} \text{ا) } y &= 2n^2 - 3n + 1 \cup \quad ns = \frac{a}{p} \rightarrow ys = \frac{2a}{18} - \frac{3a}{9} + 1 \rightarrow ys = 1 - \frac{2a}{9} = \frac{14}{9} \\ \rightarrow R &= \left[ \frac{14}{9}, +\infty \right) \end{aligned}$$

$$\begin{aligned} \text{ب) } y &= \sqrt{-n^2 + 4n - 3} \rightarrow -n^2 + 4n - 3 \geq 0 \Rightarrow ns = \frac{-4}{-2} = 2 \Rightarrow ys = -9 + 18 - 3 \\ & \quad [-\infty, 2] \rightarrow R = [0, 2] \end{aligned}$$

$$\text{ج) } y = \sqrt{2n^2 - 3n + 1} \rightarrow (-\infty, +\infty) \rightarrow R = [0, +\infty)$$

$$\text{د) } y = \log(2n^2 - 4n^2 + 4n + 1) \rightarrow n^2 - 2n^2 + 4n + 1 \rightarrow (0, +\infty) \Rightarrow R = (-\infty, +\infty)$$

$$\text{ا) } y = \frac{3n+1}{n-2} \rightarrow R = \mathbb{R} - \left\{ \frac{3}{2} \right\}$$

$$\text{ب) } y = \sqrt{\frac{3n+1}{n-2}} \rightarrow \mathbb{R} - \left\{ \frac{3}{2} \right\} \rightarrow R = [0, +\infty) - \left\{ \frac{3}{2} \right\}$$

$$\text{ج) } y = \frac{n^2 + 1}{\sqrt{n^2 + 3}} = \sqrt{n^2 + 3} + \frac{1}{\sqrt{n^2 + 3}} \rightarrow \frac{1}{\sqrt{3}} + \sqrt{3} = \frac{4\sqrt{3}}{3} \rightarrow \left[ \frac{4\sqrt{3}}{3}, +\infty \right) = \mathbb{R}$$

$$\text{د) } n^2 + \frac{1}{n^2 + 1} = y \rightarrow y = n^2 + 1 + \frac{1}{n^2 + 1} - 1 \rightarrow y = 1 + \frac{1}{n^2 + 1} - 1 = \frac{1}{n^2 + 1} \rightarrow R = \left[ \frac{1}{2}, +\infty \right)$$

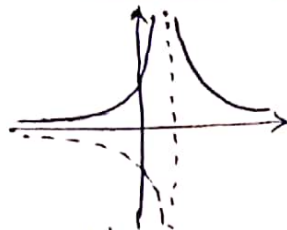
$$y = \frac{2n^2 + 1}{n^2 - 1} \rightarrow \text{درج 1} \rightarrow yn^2 - y - 2n^2 - 1 = 0 \rightarrow (y-2)n^2 + (-1-y) = 0$$

$$\Delta \geq 0 \Rightarrow 0 + 4(1+y)(y-2) \geq 0 \Rightarrow (1+y)(y-2) \geq 0 \rightarrow \frac{-1}{2} \leq y \leq 2 \rightarrow R = (-\infty, -\frac{1}{2}] \cup [2, +\infty)$$

$$\text{درج 2} \rightarrow n^2 \rightarrow [0, +\infty) \rightarrow [-1, 2] \rightarrow \text{درج 1} \rightarrow \mathbb{R} - [-1, 2]$$

$$f(x) = \frac{2 \sin x + 1}{\sin x - 2} \rightarrow -1 \leq \sin x \leq 1 \rightarrow \text{درج 1} \rightarrow [-2, \frac{1}{2}] = \mathbb{R}$$

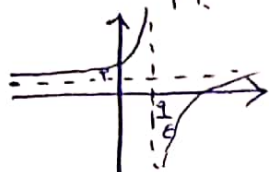
$$a) y = \left| \frac{x-1}{x} \right|$$



$$R = (0, +\infty)$$

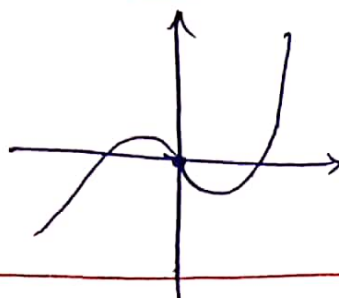
-1

$$b) \sqrt{\frac{9x-1}{x-1}}$$



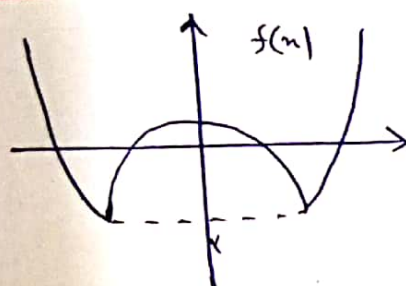
$$R = [0, +\infty) - \left\{ \frac{1}{9} \right\}$$

$$y = \begin{cases} x \geq 0 \rightarrow x^2 - 1 \\ x < 0 \rightarrow -x^2 - 1 \end{cases}$$

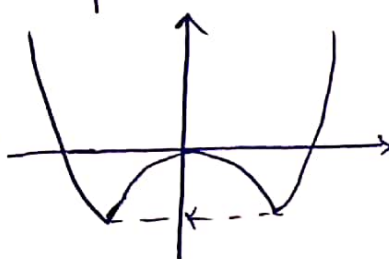


$$R = \mathbb{R}$$

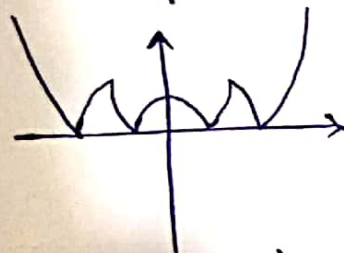
-9



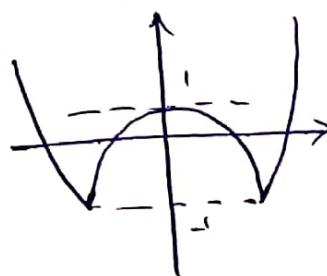
$$f(|x|)$$



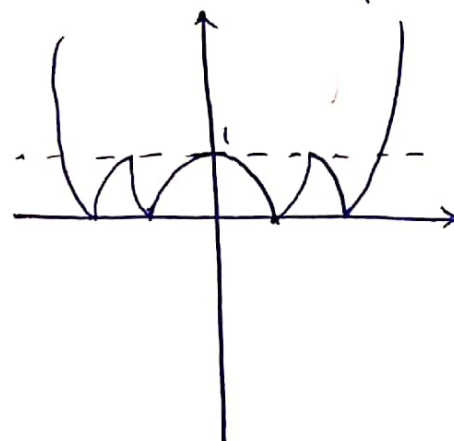
$$f(|x|) + 1$$



$$y = |f(|x|) + 1|$$



$$y = |f(|x|) + 1|$$



$$R = [0, +\infty)$$

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