

$$\begin{aligned}
 x \Rightarrow f(-x) - x f(x) &= 2|x| + 1 \Rightarrow f(-x) - x f(x) = f(x) + x f(-x) \\
 -x \Rightarrow f(x) + x f(-x) &= 2|x| + 1 \\
 \Rightarrow f(x)(-x-1) &= f(-x)(x-1) \Rightarrow f(x) \left(\frac{-x-1}{x-1} \right) = f(-x) \\
 \Rightarrow f(x) \left(\frac{x+1}{1-x} - x \right) &= 2|x| + 1 \Rightarrow \boxed{f(x) = (2|x| + 1) \left(\frac{x^2 + 1}{1-x} \right)}
 \end{aligned}$$

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$$x = 0 \Rightarrow 2f(0) = 3 \Rightarrow f(0) = \frac{3}{2}$$

$$f(x) = x^2 - 3x + 3$$

$$\boxed{f(x^2) = x^4 - 3x^2 + 3}$$

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$$y = \frac{x^2 + x + 2}{x - 1} \Rightarrow xy - y = x^2 + x + 2 \Rightarrow x^2 + (1-y)x + (y+2) = 0$$

$$\Delta: x = \frac{y-1 \pm \sqrt{y^2 - 2y + 1 - 4y - 8}}{2} \Rightarrow y^2 - 6y - 7 \geq 0 \Rightarrow y \leq -1, 7 \leq y$$

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$$R_y = (-\infty, -1] \cup [7, +\infty)$$

$$ا) \Delta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{8 - 25}{8} = \frac{-17}{8} \Rightarrow \left[\frac{-17}{8}, +\infty \right)$$

$$ب) \Delta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = 4 \Rightarrow (-\infty, 4] \Rightarrow R_y = \sqrt{(-\infty, 4]} = [0, 2]$$

$$ج) R_y = \sqrt{|R|} = [0, +\infty)$$

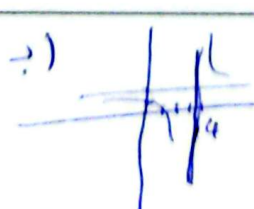
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$$د) \log(R) = \log(-\infty, +\infty) = \log(0, +\infty) = R = (-\infty, +\infty)$$

ع) محورهای مختصات



$$R_y = R - \left\{ \frac{3}{2} \right\}$$



$$R_y = \sqrt{R - \{4\}}$$

$$\Rightarrow R_y = [0, +\infty) - \{2\}$$

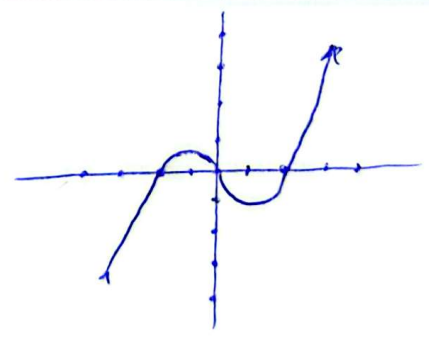
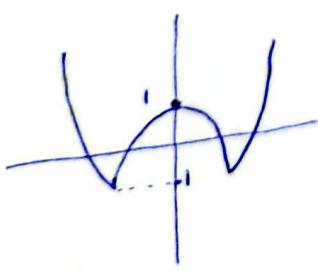
$$ح) \sqrt{x^2 + 3} = \frac{1}{\sqrt{x^2 + 3}} \Rightarrow \min(|x|) = \sqrt{3}$$

$$R_y = \left[\frac{1}{\sqrt{3}}, +\infty \right)$$

$$د) x^2 + 4 + \frac{1}{x^2 + 4} - 4 \Rightarrow$$

$$\left[\frac{17}{4}, +\infty \right) - 4 \Rightarrow R_y = \left[\frac{1}{4}, +\infty \right)$$

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$yx^2 + y = 3x^2 + 4 \Rightarrow x^2(y-3) = y+4 \Rightarrow y = \sqrt{\frac{y+4}{y-3}}$ $R_y \cup D_x = (-\infty, -4] \cup [3, +\infty)$ $\begin{array}{c c} -4 & 3 \\ \hline + & - \end{array}$ $\max y \Rightarrow x \rightarrow +\infty \Rightarrow y = 3$ $\min y \Rightarrow x = 0 \Rightarrow y = -4$ $R_y = (-\infty, -4] \cup [3, +\infty)$ چون مخرج می تواند صفر شود پس خارج دوعده	۶
$\min f(x) \Rightarrow \sin x = -1 \Rightarrow f(x) = -1$ $\max f(x) \Rightarrow \sin x = 1 \Rightarrow f(x) = -3$ $R_y = [-3, -1]$ چون ریشه های توانده صفر شود برای بین دوعده است	۷
$\text{اند } R: \left \frac{3x-1}{4x} \right \Rightarrow R - \{\frac{3}{4}\} \Rightarrow [0, +\infty) - \{\frac{3}{4}\}$ $\therefore R: \sqrt{\frac{9x-18}{4x-8}} = \sqrt{\frac{9(x-2)}{4(x-2)}} = \sqrt{\frac{9}{4}} \Rightarrow R_y = \{\frac{3}{2}\}$	۸
$x \geq 0 \Rightarrow x^2 - 2x$ $x < 0 \Rightarrow -x^2 - 2x$ $R_y = \mathbb{R}$ 	۹
 $f(x)+1$  $g(x) = f(x)+1 $	۱۰