

الف) $y = \frac{|n|+1}{|n|-1} \rightarrow |n|=t \rightarrow y = \frac{t+1}{t-1} \rightarrow y(t-1) = t+1$
 $\rightarrow t(y-1) = t+1 \rightarrow t = \frac{t+1}{y-1} \xrightarrow{t \geq 0} \frac{t+1}{y-1} \geq 0 \rightarrow R_f = (-\infty, -\frac{1}{y-1}] \cup (1, +\infty)$

ب) $y = \frac{n^2+1}{|n|} \xrightarrow{|n|=t} \frac{t^2+1}{t} = t + \frac{1}{t} = |n| + \frac{1}{|n|} \rightarrow R_f = [1, +\infty)$

$y = \frac{n^2+2n-1}{n+2} \rightarrow y(n+2) = n^2+2n-1 \rightarrow n^2+(2-y)n-(1+2y)=0$
 $\Delta \geq 0 \rightarrow (2-y)^2 - 4(1)(-1-2y) \geq 0 \rightarrow y^2+4y+12 \geq 0$
 $(y+2)(y+6) \geq 0 \rightarrow \frac{-y-2}{1+1-1+1} \rightarrow R_f = (-\infty, -6] \cup [-2, +\infty)$

$f(n) = \frac{n^2-n+\frac{1}{n}}{n^2-n+1} \rightarrow f(n) = \frac{(n-\frac{1}{n})^2}{(n-\frac{1}{n})^2+\frac{1}{n}} \xrightarrow{t=(n-\frac{1}{n})^2} f(n) = \frac{t}{t+\frac{1}{n}}$
 $\xrightarrow{t \geq 0} R_f = [0, 1)$ و $n^2-n+1 \neq 0 \rightarrow \Delta < 0 \rightarrow D_f = \mathbb{R}$

الف) $y = \sqrt{-n^2+2n-1} \xrightarrow{\Delta \leq 0} \begin{cases} n_{\max} = 1 \\ y_{\max} = 1 \end{cases} \rightarrow \sqrt{(-\infty, 1]} \rightarrow R = [0, 1]$

ب) $y = \sqrt[3]{2n^2-11n} = \sqrt[3]{2(n^2-\frac{11}{2}n)} \Rightarrow R_f = [-1, +\infty)$
 $\begin{cases} n_{\min} = 1 \\ y_{\min} = -1 \end{cases}$

الف) $\log(n^3-n+11) \rightarrow n^3-n+11 > 0 \checkmark \rightarrow R_f = \mathbb{R}$

ب) $y = \log_p\left(\frac{n^2+1}{n}\right) \rightarrow n + \frac{1}{n} > 0 \rightarrow R = [1, +\infty) \Rightarrow \log_p 1 = 1$
 $\Rightarrow R_f = [1, +\infty)$

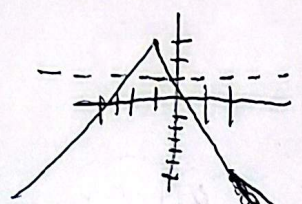
الف) $y = |2n-2| - |n-1| \rightarrow \begin{cases} n+1 & n \geq 1 \rightarrow y \geq 1 \\ 3n-2 & 1 < n < 1 \rightarrow 1 < y < 1 \\ -n-1 & n \leq 1 \rightarrow y \geq -1 \end{cases} \rightarrow R = [-1, +\infty)$



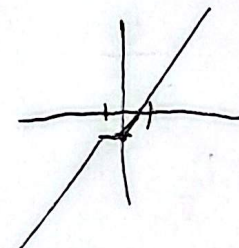
ب) $y = |n-1| + |n+1| \rightarrow R = [2, +\infty)$



$|n-1| - |2n+1| \leq 1 \rightarrow \begin{cases} -n+1 & n \geq 1 \\ -3n & -1 < n < 1 \\ n+1 & n \leq -1 \end{cases} \rightarrow \begin{cases} -n+1 & n \geq 1 \\ -3n & -1 < n < 1 \\ n+1 & n \leq -1 \end{cases} \rightarrow R = (-\infty, -1] \cup [-\frac{1}{3}, +\infty)$

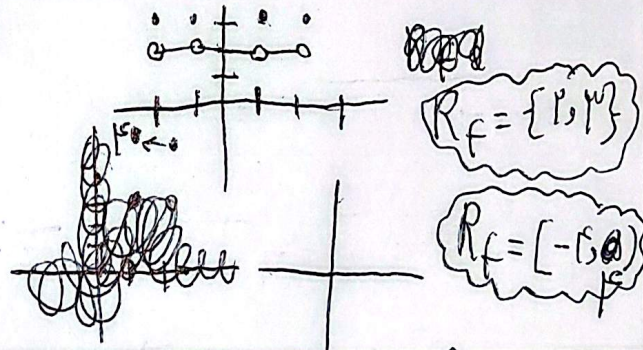


$y = 3n - |2n+1| + |n| \rightarrow \begin{cases} 2n-1 & n \geq 0 \\ 2n-1 & -\frac{1}{2} < n < 0 \\ 2n+1 & n \leq -\frac{1}{2} \end{cases} \rightarrow R_f = \mathbb{R}$



الف) $y = [n] + [1-n]$

ب) $y = [1-n] - [n]$



الف) $y = \sin^2 n - \sin n + 1 \rightarrow \begin{cases} 1 \rightarrow 1 \\ -1 \rightarrow 1 \end{cases} \Rightarrow R_f = [\frac{1}{4}, 1]$

ب) $\sin^2 n + \sin n + 1 \rightarrow \begin{cases} 1 \rightarrow 1 \\ -1 \rightarrow 1 \end{cases} \Rightarrow R = [1, 1]$