

$$1) \text{ الف) } |x| = t \Rightarrow y = \frac{t+1}{t-2} \Rightarrow y(t-2) = t+1 \Rightarrow yt - 2y = t+1 \Rightarrow t(y-1) = 2y+1$$

$$\Rightarrow t = \frac{2y+1}{y-1} \quad t \geq 0 \Rightarrow \frac{2y+1}{y-1} \geq 0 \xrightarrow{\text{ع}} \frac{-\frac{1}{2}}{+1} - \frac{1}{+1} \Rightarrow R = (-\infty, -\frac{1}{2}) \cup (1, +\infty)$$

$$ب) |x| = t \quad x \neq 0 \Rightarrow y = \frac{t^2+1}{t} \Rightarrow t + \frac{1}{t} \times t \Rightarrow yt = t^2 + 1 \Rightarrow t^2 - yt + 1 = 0$$

$$\Delta \geq 0 \Rightarrow (-y)^2 - 4(1)(1) \geq 0 \Rightarrow y^2 - 4 \geq 0 \Rightarrow (y-2)(y+2) \geq 0 \Rightarrow y \geq 2 \Rightarrow R = [2, +\infty)$$

$y=2 \quad y=-2$

$$2) \text{ طرفین ضرب کنیم} \Rightarrow y(x+2) = x^2 + 2x - 2 \Rightarrow x^2 + (2-y)x - (2+2y) = 0 \xrightarrow{\Delta \geq 0} (2-y)^2 - 4(-2-2y)$$

$$\Rightarrow y^2 - 4y + 4 + 8 + 8y \geq 0 \Rightarrow y^2 + 4y + 12 \geq 0 \Rightarrow (y+2)(y+6) \geq 0 \Rightarrow \left. \begin{matrix} y \leq -6 \\ y \geq -2 \end{matrix} \right\} R = (-\infty, -6] \cup [-2, +\infty)$$

$$3) D \Rightarrow x^2 - x + 1 = 0 \xrightarrow{\Delta} (-1)^2 - 4 = -3 < 0 \Rightarrow D = R$$

$$R \Rightarrow \text{طرفین ضرب کنیم} \Rightarrow y(x^2 - x + 1) = x^2 - x + \frac{1}{y} \Rightarrow yx^2 - yx + y = x^2 - x + \frac{1}{y}$$

$$\Rightarrow (y-1)x^2 + (1-y)x + (y - \frac{1}{y}) = 0 \xrightarrow{\Delta \geq 0} (1-y)^2 - 4(y-1)(y - \frac{1}{y}) \geq 0$$

$$\Rightarrow y^2 - 2y + 1 - (4y^2 - 2y + 4) \geq 0 \Rightarrow -3y^2 + 2y - 3 \geq 0 \Rightarrow 3y(1-y) \geq 0$$

$$\Rightarrow 0 \leq y \leq 1 \Rightarrow R = [0, 1]$$

$$4) \text{ الف) } y = \sqrt{-(x-2)^2 + 3} \Rightarrow -(x-2)^2 + 3 \geq 0 \Rightarrow 0 \leq y \leq \sqrt{3} \Rightarrow R = [0, \sqrt{3}]$$

$$ب) y = \sqrt{2(x-2)^2 - 1} \geq -1 \Rightarrow -2 \leq y < +\infty \Rightarrow R = [-2, +\infty)$$

$$5) \text{ الف) } x^3 - x + 1 \xrightarrow{x^3} \left\{ \begin{matrix} x \rightarrow -\infty \Rightarrow f(x) \rightarrow -\infty \\ x \rightarrow +\infty \Rightarrow f(x) \rightarrow +\infty \end{matrix} \right\} \xrightarrow{\text{تکامل مقدار حقیقی}} D = R$$

$$b) \frac{x^2+1}{x} = x + \frac{1}{x} \begin{cases} x > 0 \Rightarrow t = x + \frac{1}{x} \Rightarrow tx = x^2 + 1 \Rightarrow x^2 - tx + 1 = 0 \Rightarrow t^2 - 4 \geq 0 \Rightarrow t \leq -2 \vee t \geq 2 \\ x < 0 \Rightarrow t \leq -2 \end{cases}$$

$$\Rightarrow \log_2 \left(\frac{x^2+1}{x} \right) \geq \log_2 2 \Rightarrow y \geq 1 \Rightarrow R = [1, +\infty)$$

الف) $2x-2=0 \Rightarrow x=1$
 $x-3=0 \Rightarrow x=3$
 $\Rightarrow x=1 \Rightarrow |0|-|-2|=-2$

$$x=3 \Rightarrow |4|-|0|=4, x=0 \Rightarrow y=-1, x=4 \Rightarrow y=5$$

ب) $x-3=0 \Rightarrow x=3$

$$x+1=0 \Rightarrow x=-1$$

ج) $x=3 \Rightarrow y=4, x=-1 \Rightarrow y=4, x=4 \Rightarrow y=5, x=-2 \Rightarrow y=-4$

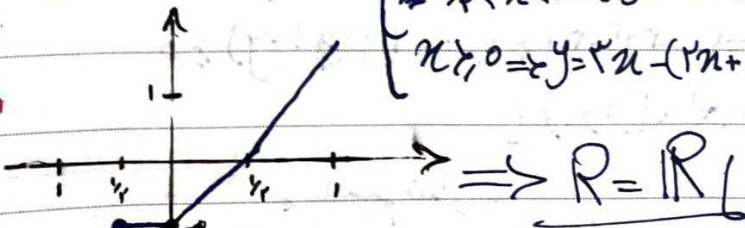
$$x=-1 \Rightarrow x < -1 \Rightarrow (2-x) - (-2x-2) \leq 1 \Rightarrow x+4 \leq 1 \Rightarrow (-\infty, -3]$$

$$-1 < x < 2 \Rightarrow (2-x) - (2x+2) \leq 1 \Rightarrow x > -\frac{1}{3} \Rightarrow [-\frac{1}{3}, 2)$$

$$x \geq 2 \Rightarrow (x-2) - (2x+2) \leq 1 \Rightarrow x \geq -5$$

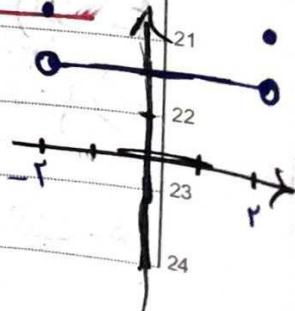
$$\Rightarrow (-\infty, -3] \cup [-\frac{1}{3}, +\infty)$$

$$A - \frac{0}{2} = x = -\frac{1}{2} \Rightarrow \begin{cases} x < -\frac{1}{2} \Rightarrow y = 3x - (2x-1) - x = x+1 \Rightarrow x = -\frac{1}{2}, y = -\frac{1}{2} \\ -\frac{1}{2} < x < 0 \Rightarrow y = 3x - (2x+1) - x = -1 \\ x > 0 \Rightarrow y = 3x - (2x+1) + x = x-1 \Rightarrow x = 0, y = -1 \end{cases}$$



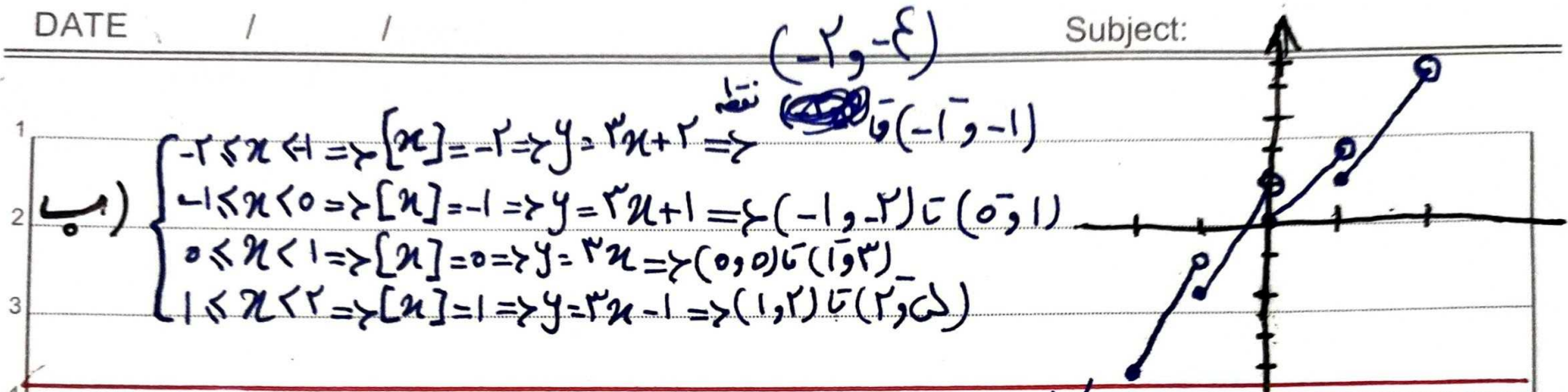
$$\Rightarrow R = \mathbb{R}$$

الف) $-2 \leq x < -1 \Rightarrow [x] = -2, 2-2x \leq 4 \Rightarrow [2-x] = 4 \Rightarrow y = -2+4 = 2$
 $-1 \leq x < 0 \Rightarrow [x] = -1, 2-2x \leq 4 \Rightarrow [2-x] = 4 \Rightarrow y = -1+4 = 3$
 $0 \leq x < 1 \Rightarrow [x] = 0, 2-2x \leq 4 \Rightarrow [2-x] = 2 \Rightarrow y = 0+2 = 2$
 $1 \leq x < 2 \Rightarrow [x] = 1, 2-2x \leq 4 \Rightarrow [2-x] = 1 \Rightarrow y = 1+1 = 2$



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Subject:



الف- ١٥) $\sin x = t \Rightarrow y = t^2 - t + 1$ $0 \leq t \leq 1$

$$y = \left(t - \frac{1}{2}\right)^2 + \frac{3}{4}$$

مع كمال $y \geq \frac{3}{4}$

مع صغ $y \leq \frac{3}{4}$

Max $t = -1 \rightarrow y = 1 + 1 + 1 = 3 \Rightarrow R = \left[\frac{3}{4}, 3\right]$

ب) $t = \sin^2 x \Rightarrow y = t^2 + t + 1$ $0 \leq t \leq 1$

$$\left. \begin{aligned} t = 0 &\Rightarrow y = 1 \\ t = 1 &\Rightarrow y = 3 \end{aligned} \right\} \Rightarrow R = [1, 3]$$