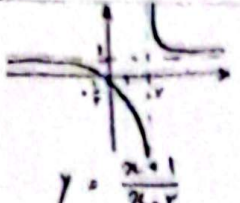
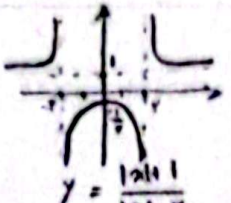


الف) $y = \frac{|x|+1}{|x|-x}$   $R_f = IR - (-\frac{1}{x} + 1]$

ب) $y = \frac{x^2+1}{|x|}$ $x > 0 \Rightarrow y = \frac{x^2+1}{x} = x + \frac{1}{x} \Rightarrow y \geq 2$ $x < 0 \Rightarrow y = \frac{x^2+1}{-x} = -(\frac{1}{x} + x) \Rightarrow y \leq -2$ $R_f = [2, +\infty) \cup (-\infty, -2]$

$y = \frac{x^2+2x-2}{x+2}$ $xy + 2y = x^2 + 2x - 2 \Rightarrow x^2 + (2-y)x - (2y+2) = 0$

$x = \frac{y-2 \pm \sqrt{(y-2)^2 + 4(2y+2)}}{2}$ $y^2 - 4y + 4 + 8y + 8 \geq 0 \Rightarrow y^2 + 4y + 12 \geq 0$

$(y+4)(y+3) \geq 0 \Rightarrow y \leq -4 \text{ or } y \geq -3$ $R_f = IR - (-4, -3)$

$f(x) = \frac{x^2-x+\frac{1}{x}}{x^2-x+1}$ $f(x) = \frac{(x-\frac{1}{x})^2}{(x-\frac{1}{x})^2+\frac{2}{x}} = y$ $(x-\frac{1}{x})^2 = t \Rightarrow y = \frac{t}{t+\frac{2}{x}}$

$yt + \frac{2}{x}y = t \Rightarrow \frac{y}{t}y = (1-y)t \Rightarrow t = \frac{xy}{t-ty}$ $(x-\frac{1}{x})^2 = \frac{xy}{t-ty}$

$x - \frac{1}{x} = \pm \sqrt{\frac{xy}{t-ty}} \Rightarrow x = \pm \sqrt{\frac{xy}{t-ty}} + \frac{1}{x}$ $\frac{xy}{t-ty} \geq 0 \Rightarrow -\frac{1}{4} + \frac{1}{4} = 0$

$R_f = [0, 1]$

الف) $y = \sqrt{-x^2+4x-1}$ $y^2 = -x^2+4x-1 \Rightarrow x^2-4x+1+y^2=0$

$x = \frac{4 \pm \sqrt{16-4(1+y^2)}}{2} = 2 \pm \sqrt{3-y^2}$ $16-4(1+y^2) \geq 0 \Rightarrow 4-y^2 \geq 0 \Rightarrow y^2 \leq 4 \Rightarrow -2 \leq y \leq 2$

$R_f = [0, \sqrt{3}]$

ب) $y = \sqrt[3]{x^2+4x}$ $x^2+4x = y^3 \Rightarrow x^2+4x-y^3=0 \Rightarrow x = \frac{-4 \pm \sqrt{16+4y^3}}{2}$

$4 \pm \sqrt{16+4y^3} \geq 0 \Rightarrow \sqrt{16+4y^3} \geq -4 \Rightarrow y^3 \geq -16 \Rightarrow y \geq -2$ $R_f = [-2, +\infty)$

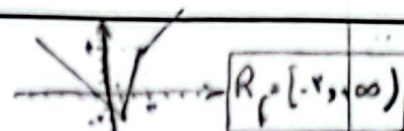
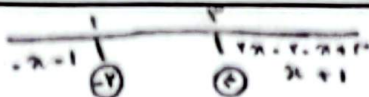
الف) $y = \log(x^2-x+1)$ $-\infty < x^2-x+1 < +\infty \Rightarrow \frac{\log}{x^2-x+1} \Rightarrow -\infty < \log(x^2-x+1) < +\infty$

$R_f = IR$

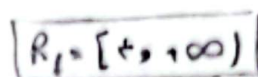
ب) $y = \log(\frac{x^2+1}{x})$ $\frac{x^2+1}{x} = x + \frac{1}{x} \geq 2 \text{ for } x > 0$ $\frac{\log}{x} \Rightarrow \log(\frac{x^2+1}{x}) \geq 0$

$R_f = [1, +\infty)$

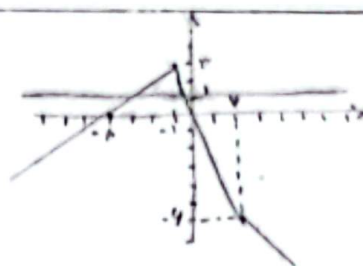
الف) $y = |2x-2| - |x-2|$



ب) $y = |x-2| + |x+1|$ نقطه صفر



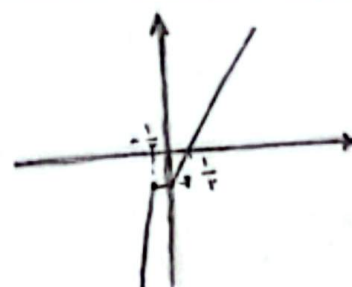
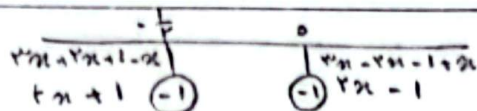
$y = |x-2| - |3x+2|$



$-3x \leq 1 \Rightarrow x \geq -\frac{1}{3}$
 $x+2 \leq 1 \Rightarrow x \leq -1$

$x \in \mathbb{R} - (-\frac{1}{3}, -1]$

$y = 3x - |2x+1| + |x|$



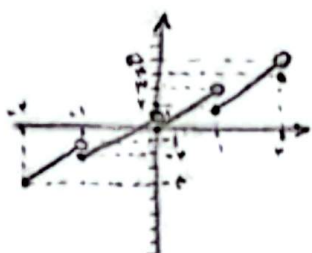
$R_f = \mathbb{R}$

الف) $y = [x] + [2-x]$ $y = [x] + [-x] + 2$



$R_f = \{2, 3\}$

ب) $y = 3x - [x]$



$R_f = [-1, \infty)$

الف) $y = \sin^2 x - \sin x + 1$

$\max \sin x = 1 \Rightarrow y = 1 - 1 + 1 = 1$
 $\min \sin x = -1 \Rightarrow y = 1 + 1 + 1 = 3$

$R_f = [\frac{3}{2}, 3]$

ب) $y = \sin^2 x + \sin^2 x + 1$

$\max \sin^2 x = 1 \Rightarrow y = 1 + 1 + 1 = 3$
 $\min \sin^2 x = 0 \Rightarrow y = 1$

$R_f = [1, 3]$