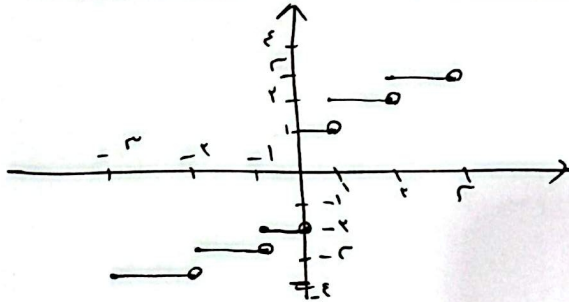


الف)  $0 \leq \sin^2 \alpha \leq 1 \rightarrow 0 \leq 2 \sin^2 \alpha \leq 2 \rightarrow -1 \leq f(\alpha) \leq 1$

$R_f = [-1, 1]$

ب)  $-1 \leq \cos \alpha \leq 1 \rightarrow -2 \leq \cos^2 \alpha \leq 2 \rightarrow -2 \leq 2 \cos^2 \alpha \leq 2$

$\rightarrow -4 \leq 2 \cos^2 \alpha - 2 \leq -1 \rightarrow [-4, -1]$



$f(x) \rightarrow -2 \rightarrow -2$

$f(x) \rightarrow -1 \rightarrow -1$

$f(x) \rightarrow 0 \rightarrow 0$

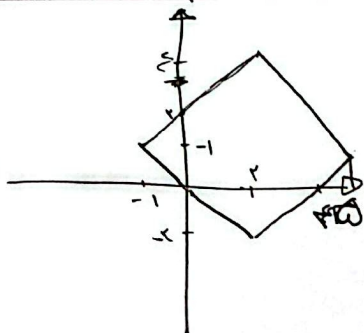
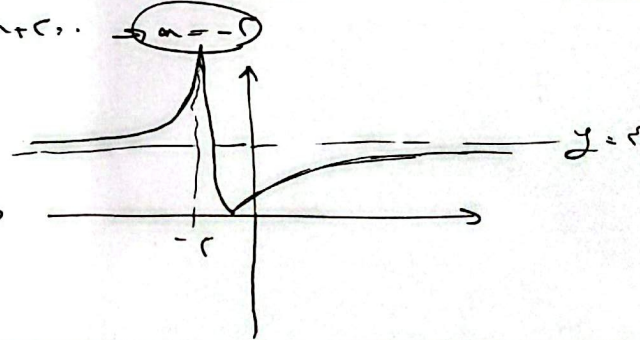
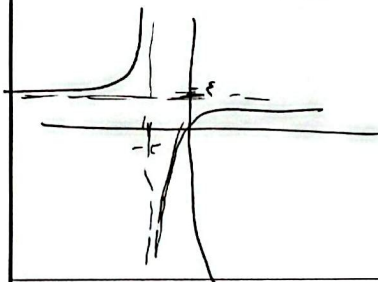
$f(x) \rightarrow 1 \rightarrow 1$

$f(x) \rightarrow 2 \rightarrow 2$

$f(x) \rightarrow 2$

max  $\frac{x}{1} = 2$

$\alpha + 2 \rightarrow \alpha = -2$



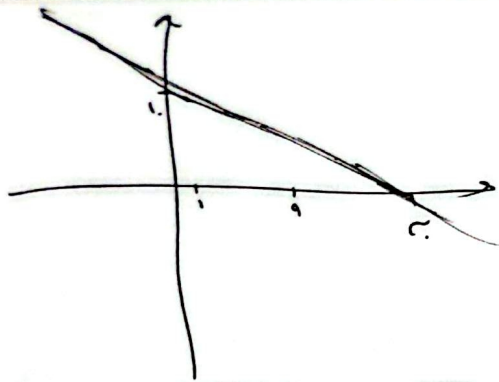
$$y = \begin{cases} \alpha + 2 & -1 \leq \alpha \leq 2 \\ 4 - \alpha & 2 \leq \alpha \leq 4 \\ -\alpha & -1 \leq \alpha \leq 2 \\ \alpha - 2 & 2 \leq \alpha \leq 4 \end{cases}$$

الف)  $\alpha + \frac{1}{\alpha} = z \rightarrow z \geq 0 \quad \alpha + \frac{1}{\alpha} \in (-\infty, -2] \cup [2, +\infty)$

$z \geq 0 \rightarrow z \in [2, +\infty) \quad f(z) = 2 + 2\sqrt{z-1} \quad z \geq 2 \rightarrow R = [2 + 2\sqrt{2-1}, +\infty)$

ب)  $z = \sin \alpha \rightarrow -1 \leq z \leq 1 \rightarrow (2-z)(1+z) = 2 + 2z - z^2$

$y = 2 - (z-1)^2 \rightarrow 1 \leq z \leq 1 \rightarrow y_{\max} = 2 \quad z = 1 \rightarrow y = 2$   
 $R_y = [0, 2]$



$$\begin{aligned} f(0) &= 1 \\ f(1) &= 0 \\ f(2) &= -1 \\ f(3) &= -2 \\ f(4) &= -3 \end{aligned}$$

$$\begin{aligned} f(0) &= 1 \\ f(1) &= 0 \\ f(2) &= -1 \\ f(3) &= -2 \\ f(4) &= -3 \end{aligned}$$

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$$\begin{aligned} a m^2 + b m + c &= a(m-1)^2 \rightarrow a m^2 + b m + c = a m^2 - 2 a m + a \\ b &= -2a \quad c = a \quad f(0) = 1 \quad \sqrt{9a - 4a + 4a} = 1 \rightarrow \sqrt{a} = 1 \\ a &= 1 \\ a &= 1 \quad b = -2 \quad c = 1 \quad y(m) = \sqrt{-m^2 + m - 1} \rightarrow -m^2 + m - 1 \\ -2\left(\frac{1}{2}\right)^2 + \frac{1}{2} - 1 &= -\frac{1}{4} \quad R_f = \emptyset \end{aligned}$$

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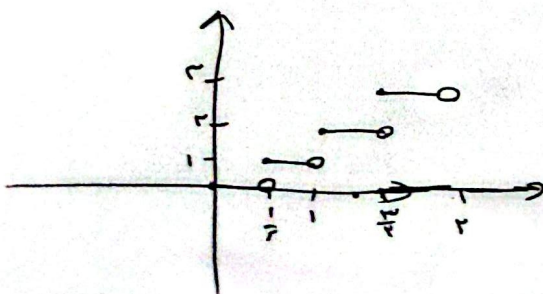
$$\begin{aligned} n(m^2 - 1) &\rightarrow n(m+1)(m-1) \rightarrow f(m) = n(m-1) = m^2 - 2m \\ f(m) &= m^2 - 2m = (m-1)^2 - 1 \xrightarrow{1/4} [-1, 1] \quad m \neq -1 \quad f(-1) \neq 1 \\ (m-1)^2 - 1 &= 1 \rightarrow (m-1)^2 = 2 \rightarrow m = 1 \pm \sqrt{2} \quad \{a, b\} = \{-1, 1\} \\ a + b &= 2 \quad c = -1 \quad f\left(\frac{a+b}{2}\right) = f(0) = f(-1) = 1 \end{aligned}$$

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$$\begin{aligned} (m-1)^2 &\rightarrow m^2 - 2m + 1 \rightarrow a = 1 \quad b = -2 \\ y &= \sqrt{(m-1)^2} \rightarrow y = m-1 \quad [a, b] = [-1, 1] \\ f(-1) &= -1 \quad R = [-1, 1] \\ f(1) &= 1 \end{aligned}$$

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$$\begin{aligned} f(x) &= 0, \quad x \in [0, 1/2] \\ f(x) &= 1, \quad x \in (1/2, 1] \end{aligned}$$

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