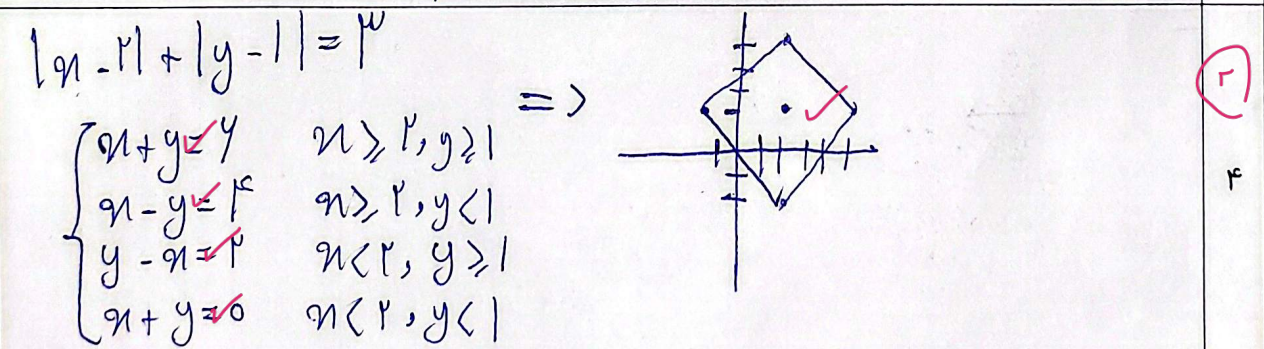
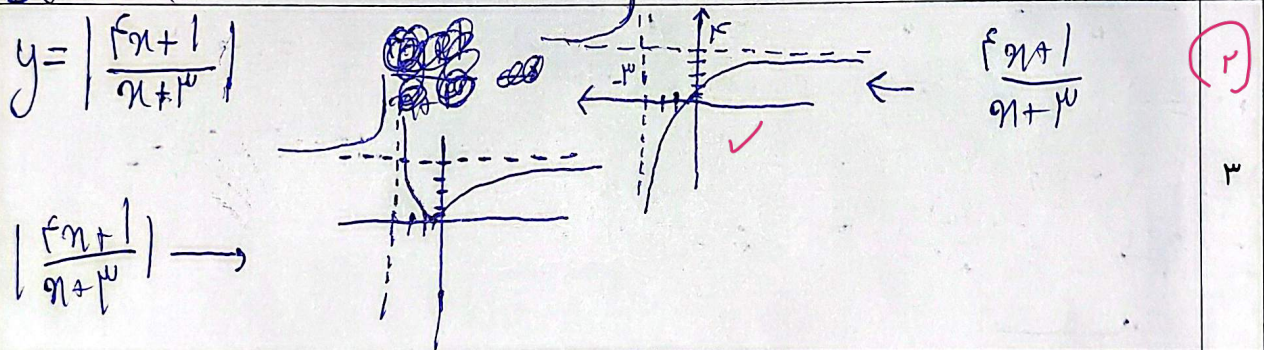
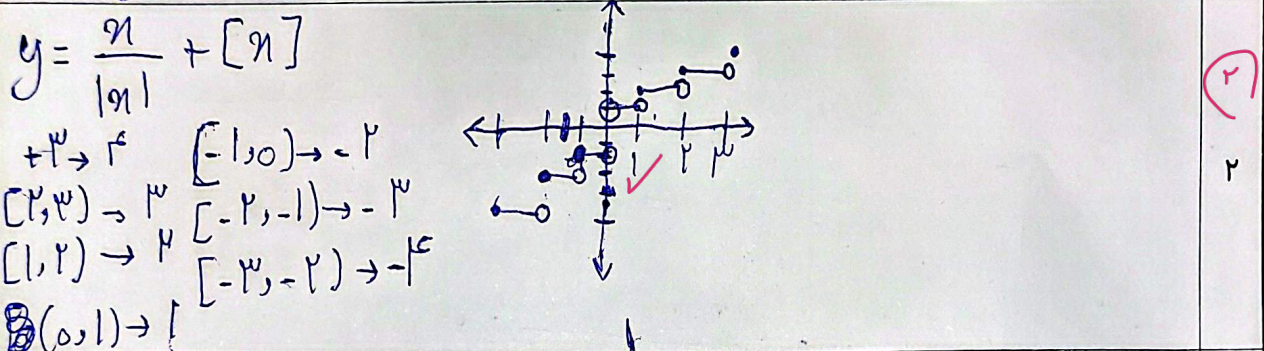


الف) $f(x) = 1 \sin^2(x + \frac{\pi}{3}) - 1 \rightarrow -1 \leq \sin(x + \frac{\pi}{3}) \leq 1$
 $\rightarrow 0 \leq \sin^2(x + \frac{\pi}{3}) \leq 1 \leftrightarrow 0 \leq 1 \sin^2(x + \frac{\pi}{3}) \leq 1 \rightarrow -1 \leq 1 \sin^2(x + \frac{\pi}{3}) - 1 \leq 1$
 $\rightarrow R_f = [-1, 1]$

ب) $g(x) = 1 \cos^3 x - 1 \rightarrow -1 \leq \cos^3 x \leq 1 \rightarrow -1 \leq 1 \cos^3 x \leq 1 \rightarrow R_f = [-1, 1]$



الف) $f(x) = x + \frac{1}{x} + 1 \sqrt{x + \frac{1}{x}}$ $x + \frac{1}{x} = t \Rightarrow x + \frac{1}{x} \leq -2, x + \frac{1}{x} \geq 2$
 $f = t + 1 \sqrt{t} \quad t \geq 2 \Rightarrow t \geq 2 \Rightarrow f_{min} = 2 + 1 \sqrt{2} \Rightarrow R_f = [2 + 1 \sqrt{2}, +\infty)$

ب) $y = (3 - \sin x)(1 + \sin x) \rightarrow -2 \leq \sin x \leq 2 \leftarrow \begin{matrix} (3-1)(1+1) = 4 \\ (3-1)(1-1) = 0 \end{matrix} \rightarrow R = [0, 4]$

$$f(n) = -\frac{1}{p}n + 1 \quad D_f = [1, 9] \quad \forall n \in \mathbb{N}$$

$$f(1) = \frac{p}{p}, f(9) = 0 \rightarrow S = \frac{9(\frac{p}{p} + 0)}{p} = \frac{9(\frac{p+0}{p})}{p} = \boxed{\frac{9}{p}}$$

$$f(n) = \sqrt{an^p + bn + c}, n \geq p \quad an^p + bn + c = a(n^p)^p = a n^{p^2} - f_a n + f_a$$

$$\rightarrow b = -f_a, c = f_a \quad f(p) = \sqrt{9a + p b + c} = 1 \rightarrow f(p) = \sqrt{9a - p f_a + f_a} = 1$$

$$\sqrt{a} = 1 \rightarrow a = 1 \quad a = 1, b = -f, c = f \quad g(n) = \sqrt{-f n^p + n - 1}$$

$$-f n^p + n - 1 \geq 0 \quad \Delta = -1^2 V < 0 \Rightarrow R_g = \emptyset$$

$$f(n) = \frac{n^p - f n}{n + p} = \frac{n(n^p - f)}{n + p} = \frac{n(n - p)(n + p)}{n + p} = n(n - p)$$

$$\rightarrow n \neq -p, n(n - p) \neq 0 \rightarrow n^p - p n = \Lambda \neq 0 \quad (n - p)(n + p) \neq 0 \rightarrow n \neq p, n \neq -p$$

$$D_f = \mathbb{R} - \{a, b\} \rightarrow a = -p, b = p \quad n^p - p n \rightarrow \frac{n^p - p n}{n + p} \Rightarrow c = -1$$

$$f\left(\frac{a+b}{p}\right) = f\left(\frac{-p+p}{p}\right) = f(-1) \Rightarrow f(-1) = -1(-p) = p$$

$$y = \sqrt[p]{n^p + a n^p + b n - \Lambda} \quad [a, b]$$

$$\hookrightarrow (n - p)^p = n^p - p n^p + p n - \Lambda \Rightarrow a = -p, b = p$$

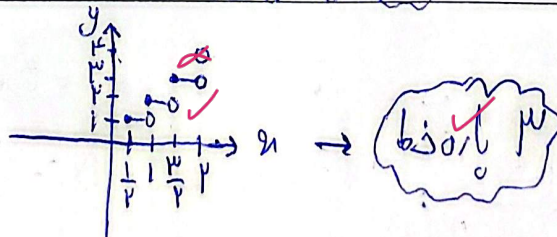
$$f(n) = \sqrt[p]{(n - p)^p} = n - p \quad f(-4) = -1, f(12) = 10$$

$$\Rightarrow R_f = [-1, 10]$$

$$\left[\frac{1}{p}, 1\right) \rightarrow y = 1$$

$$\left[1, \frac{p}{p}\right) \rightarrow y = p$$

$$\left[\frac{p}{p}, p\right) \rightarrow y = p^p$$



۷- با توجه به دامنه، زیر رادیکال یک خط با شیب مثبت بوده یعنی $a=0$ است و نقطه $(2,0)$ هم در معادله خط صدق می‌کند...

$$\begin{cases} (2,0) \rightarrow 2b+c=0 \\ (3,1) \rightarrow 3b+c=1 \end{cases} \rightarrow b=1 \text{ و } c=-2$$

$$g(x) = \sqrt{x^2+4} \rightarrow \text{ext} \Big|_4^0 \Rightarrow R_g = \sqrt{[4, +\infty)} = [2, +\infty)$$