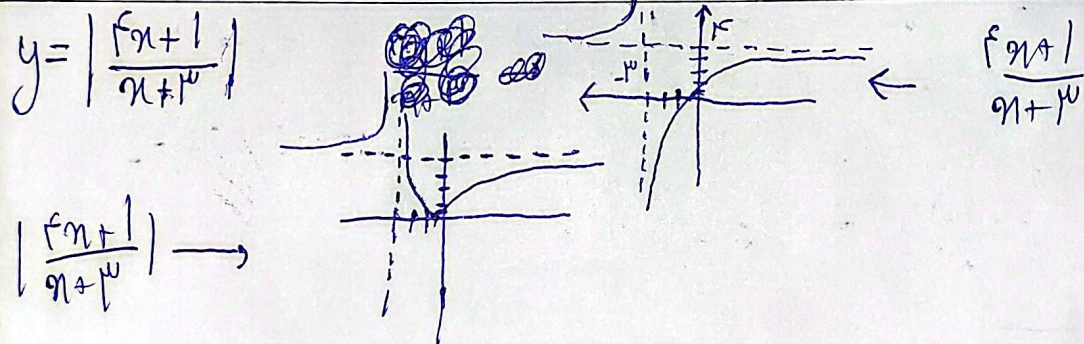
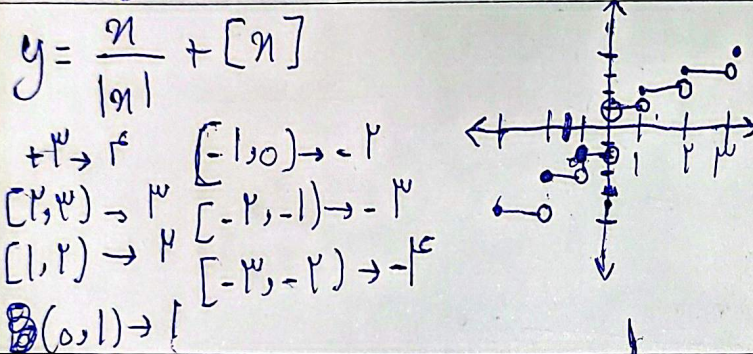


الف) $f(x) = 2 \sin^2(x + \frac{\pi}{3}) - 1 \rightarrow -1 \leq \sin(x + \frac{\pi}{3}) \leq 1$
 $\rightarrow 0 \leq \sin^2(x + \frac{\pi}{3}) \leq 1 \leftrightarrow 0 \leq 2 \sin^2(x + \frac{\pi}{3}) \leq 2 \rightarrow -1 \leq 2 \sin^2(x + \frac{\pi}{3}) - 1 \leq 1$
 $\rightarrow R_f = [-1, 1]$

ب) $g(x) = 2 \cos^3 x - 1 \rightarrow -1 \leq \cos x \leq 1 \rightarrow -1 \leq \cos^3 x \leq 1 \rightarrow R_f = [-1, 1]$



$|x-2| + |y-1| = 3$

$\Rightarrow$

$$\begin{cases} x+y=4 & x \geq 2, y \geq 1 \\ x-y=4 & x \geq 2, y < 1 \\ y-x=4 & x < 2, y \geq 1 \\ x+y=0 & x < 2, y < 1 \end{cases}$$

الف) $f(x) = x + \frac{1}{x} + 2\sqrt{x + \frac{1}{x}}$ $x + \frac{1}{x} = t \Rightarrow x + \frac{1}{x} \leq -2, x + \frac{1}{x} \geq 2$
 $f = t + 2\sqrt{t} \quad t \geq 2 \Rightarrow f_{min} = 2 + 2\sqrt{2} \Rightarrow R_f = [2 + 2\sqrt{2}, +\infty)$

ب) $y = (3 - \sin x)(1 + \sin x) \rightarrow -2 \leq \sin x \leq 2$
 $(3-1)(1+1) = 4$
 $(3-1)(1-1) = 0 \rightarrow R = [0, 4]$

$$f(x) = -\frac{1}{p}x + 1 \quad D_f = [1, 9] \quad g \in \mathbb{N}$$

$$f(1) = \frac{p}{p}, f(9) = 0 \quad \rightarrow S = \frac{g(\frac{p}{p}, V)}{p} = \frac{g(\frac{p}{p})}{p} = \underline{V \Delta}$$

$$f(x) = \sqrt{ax^p + bx + c}, x \geq p \quad ax^p + bx + c = a(x-p)^p = ax^p - p_a x + p_a$$

$$\rightarrow b = -p_a, c = p_a$$

$$f(p) = \sqrt{9a + p_b + c} = 1 \rightarrow f(p) = \sqrt{9a - p_a + p_a} = 1$$

$$\sqrt{a} = 1 \rightarrow a = 1$$

$$a = 1, b = -p, c = p$$

$$g(x) = \sqrt{-fx^p + x - 1}$$

$$-fx^p + x - 1 \geq 0$$

$$\Delta = -1^2 V < 0 \Rightarrow R_g = \emptyset$$

$$f(x) = \frac{x^p - fx}{x+p} = \frac{x(x^p - f)}{x+p} = \frac{x(x-p)(x+p)}{x+p} = x(x-p)$$

$$\rightarrow x \neq -p, x(x-p) \neq 0 \rightarrow x^p - px - 1 \neq 0 \quad (x-p)(x+p) \neq 0 \rightarrow x \neq p, x \neq -p$$

$$D_f = \mathbb{R} - \{a, b\} \rightarrow a = -p, b = p$$

$$x^p - px \rightarrow \cancel{x} \Rightarrow c = -1$$

$$f\left(\frac{a+b}{p}\right) = f\left(\frac{p-p}{p}\right) = f(-1) \Rightarrow f(-1) = -f(-p) = p$$

$$y = \sqrt[p]{x^p + ax^p + bx - 1} \quad [a, b]$$

$$\hookrightarrow (x-p)^p = x^p - px^p + 1px - 1 \Rightarrow a = -p, b = 1p$$

$$f(x) = \sqrt[p]{(x-p)^p} = x - p \quad f(-p) = -1, f(1p) = 1$$

$$\Rightarrow R_f = [-1, 1]$$

$$\left[\frac{1}{p}, 1\right) \rightarrow y = 1$$

$$\left[1, \frac{p}{p}\right) \rightarrow y = p$$

$$\left[\frac{p}{p}, p\right) \rightarrow y = p$$

