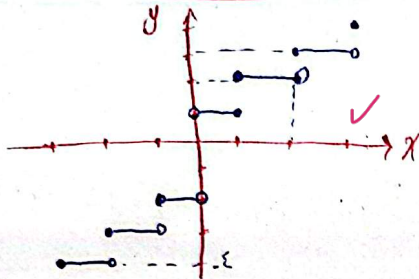
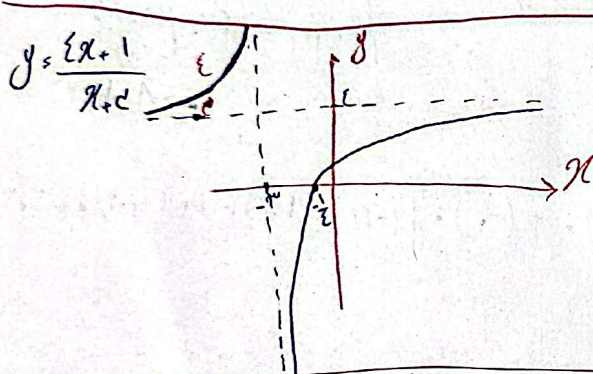


الف) $f(x) = 2 \sin^2(x + \frac{\pi}{4}) - 1 \Rightarrow \sin^2(x + \frac{\pi}{4}) \in [0, 1] \Rightarrow 2 \sin^2(x + \frac{\pi}{4}) \in [0, 2] \Rightarrow -1(2 \sin^2(x + \frac{\pi}{4}) - 1) \in [-1, 1]$
 $R_f = [-1, 1]$

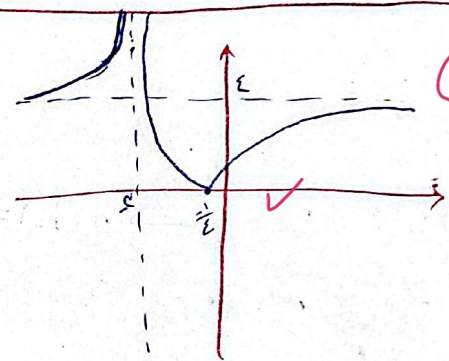
ب) $g(x) = 3 \cos^2 x - 4 \Rightarrow \cos^2 x \in [0, 1] \Rightarrow 3 \cos^2 x \in [0, 3] \Rightarrow -1(3 \cos^2 x - 4) \in [-3, 4]$
 $R_f = [-3, 4]$



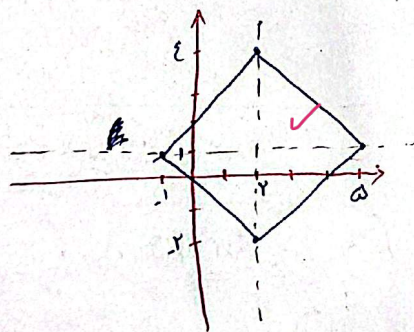
$-1 \cdot 3 = -3 \leftarrow x \in [-2, -1]$
 $-1 \cdot 2 = -2 \leftarrow x \in [-1, 0]$
 $-1 \cdot 1 = -1 \leftarrow x \in [0, 1]$
 $1 + 0 = 1 \leftarrow x \in [1, 2]$
 $1 + 1 = 2 \leftarrow x \in [2, 3]$
 $1 + 2 = 3 \leftarrow x \in [3, 4]$
 $1 + 3 = 4 \leftarrow x \in [4, 5]$



$y = \left| \frac{x+1}{x+2} \right|$



$|x-2| + |y-1| = 3$



۴- ابتدا ریشه‌ها را بدست آورده و نقطه برخورد را محاسبه کنیم

$x - y + y - 1 = 3 \Rightarrow y = -x + 4$
 $-x + y + y - 1 = 3 \Rightarrow y = x + 2$
 $x - y - y + 1 = 3 \Rightarrow y = x - 2$
 $-x + y - y + 1 = 3 \Rightarrow y = -x$

الف) $f(x) = x + \frac{1}{x} + 2\sqrt{x + \frac{1}{x}} \Rightarrow x + \frac{1}{x} > 0 \Rightarrow x > 0$
 $\Rightarrow \sqrt{x + \frac{1}{x}} \in [\sqrt{2}, +\infty) \Rightarrow 2\sqrt{x + \frac{1}{x}} \in [2\sqrt{2}, +\infty)$
 $R_f = [2 + 2\sqrt{2}, +\infty)$

ب) $y = (3 - \sin x)(1 + \sin x) = -\sin^2 x + 2\sin x + 3$

$R_y = [0, 4]$

$\max \sin x \Rightarrow \max \sin x = 1 \Rightarrow -\sin^2 x + 2\sin x + 3 = -1 + 2 + 3 = 4$

$\min \sin x \Rightarrow \min \sin x = -1 \Rightarrow -\sin^2 x + 2\sin x + 3 = -1 - 2 + 3 = 0$

$x = \frac{\pi}{2} \Rightarrow \frac{\pi}{2} = \frac{\pi}{2} \Rightarrow -\sin^2 x + 2\sin x + 3 = -1 + 2 + 3 = 4$

$$f(x) = \frac{1}{c}x + 1$$

اندازه طبیعی

$$یک مجموعه = \{1, 2, \dots, 9\}$$

$$(1+2+\dots+9) = \frac{9 \times 10}{2} = 45$$

(2) -9

$$\Rightarrow \frac{1}{3}(1+2+\dots+9) + 1 \times 9 = \frac{1}{3}(45) + 9 = 15 + 9 = 24 \quad (\checkmark)$$

$$2 \text{ یا } 3, x=5 \Rightarrow \frac{1}{3}(5) + 1 = \frac{5}{3} + 1 = \frac{8}{3} \quad \frac{24}{3} \times 3 = 24 \times 3 = 72 \quad (\checkmark)$$

$$f(x) = \sqrt{ax^2 + bx + c}$$

فقط بازه $[2, +\infty)$ تعریف می شود $\Leftarrow$ خطی است بازه $2 \leq x$

$$\left. \begin{aligned} f(2) = \sqrt{4b+c} = 0 &\Rightarrow 4b+c=0 \\ f(3) = \sqrt{9b+c} = 1 &\Rightarrow 9b+c=1 \end{aligned} \right\} \Rightarrow \begin{aligned} b &= 1 \\ c &= -4 \end{aligned}$$

$$g(x) = \sqrt{x^2 + 4} \Rightarrow \mathbb{R} \setminus \{x \mid x < -2\} \Rightarrow R = [2, +\infty)$$

$$\hookrightarrow R_{g(x)} = \sqrt{[2, +\infty)} = [2, +\infty)$$

$$f(x) = \frac{x^2 - 4x}{x+2} = \frac{x(x-4)}{x+2} = \frac{x(x+2)(x-2)}{x+2} = x^2 - 2x \quad ; \quad x \neq -2$$

$$x^2 - 2x \neq 1 \Rightarrow x^2 - 2x - 1 \neq 0 \Rightarrow (x-1)(x+2) \neq 0 \Rightarrow x \neq -2, x \neq 1$$

$$D_f = \mathbb{R} \setminus \{-2, 1\}$$

$$R_f = [-1, +\infty) \setminus \{1\} \Rightarrow c = -1 \Rightarrow 2c = -2 \Rightarrow f\left(\frac{a+b}{2c}\right) = f\left(\frac{-2}{-2}\right) = f(1) = 1 - 2(-1) = 1 + 2 = 3$$

$$\hookrightarrow f(1) = 1 - 2 = -1$$

$$y = \sqrt[3]{x^3 + ax^2 + bx - 1} = \sqrt[3]{(x-K)^3}$$

خطی است پس بصورت $\sqrt[3]{(x-K)^3}$ نوشته می شود

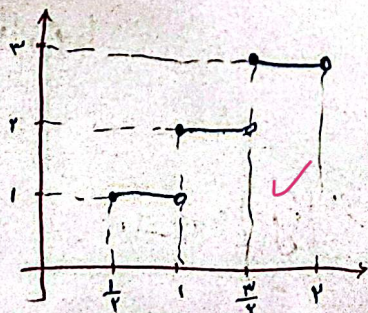
$$-1 = -K^3 \Rightarrow K^3 = 1 \Rightarrow K = 1$$

$$\sqrt[3]{(x-1)^3} = \sqrt[3]{x^3 - 3x^2 + 3x - 1} \quad \begin{cases} a = -3 \\ b = 3 \end{cases}$$

$$D_f = [-1, 1]$$

$$\sqrt[3]{(x-1)^3} = (x-1) \quad \begin{cases} x=1 \Rightarrow -4-2=-1 \\ x=0 \Rightarrow 1-2=-1 \end{cases}$$

$$\Rightarrow R_f = [-1, 1]$$



$$y = \begin{cases} 0+1=1 & ; \quad \frac{1}{4} \leq x < 1 \\ 1+1=2 & ; \quad 1 \leq x < \frac{3}{4} \\ 2+1=3 & ; \quad \frac{3}{4} \leq x < 2 \end{cases}$$

بازه ها را بنویس

(2) -4