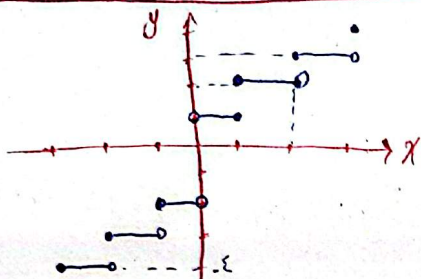


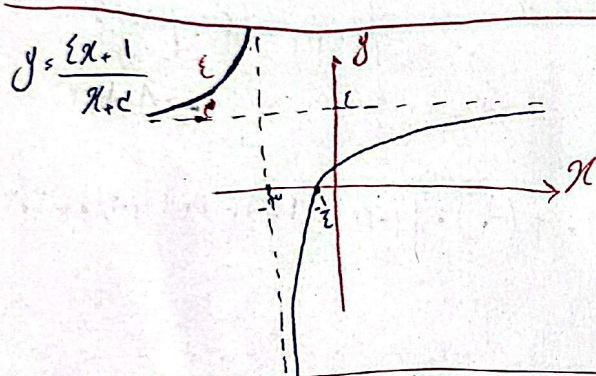
الف) $f(x) = 2 \sin^2(x + \frac{\pi}{4}) - 1 \Rightarrow \sin^2(x + \frac{\pi}{4}) \in [0, 1] \Rightarrow 2 \sin^2(x + \frac{\pi}{4}) \in [0, 2] \Rightarrow -1 \leq 2 \sin^2(x + \frac{\pi}{4}) - 1 \leq 1$

$$R_f = [-1, 1]$$

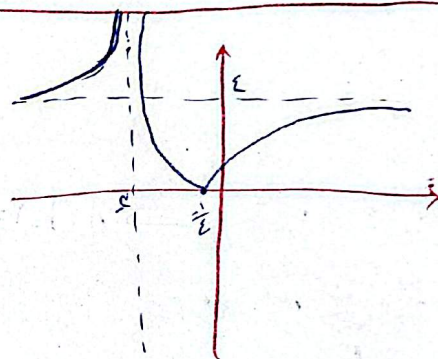
ب) $g(x) = 3 \cos^2 x - 4 \Rightarrow \cos^2 x \in [0, 1] \Rightarrow 3 \cos^2 x \in [0, 3] \Rightarrow -4 \leq 3 \cos^2 x - 4 \leq -1 \Rightarrow R_g = [-4, -1]$



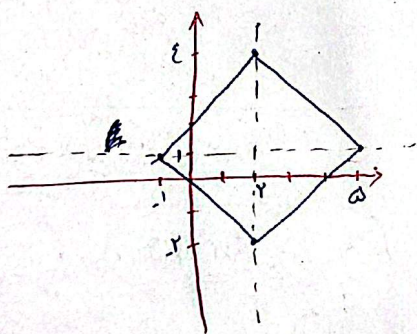
$$\begin{aligned} -1 \leq x \leq -2 &\leftarrow -1 \\ -2 \leq x \leq -1 &\leftarrow -2 \\ -1 \leq x \leq 0 &\leftarrow -1 \\ 0 \leq x \leq 1 &\leftarrow 0 \\ 1 \leq x \leq 2 &\leftarrow 1 \\ 2 \leq x \leq 2 &\leftarrow 2 \\ 2 \leq x \leq 2 &\leftarrow 1 \end{aligned}$$



$$y = \left| \frac{x+1}{x+2} \right|$$



$$|x-2| + |y-1| = 3$$



ع- ابتدا ریشه‌ها را درست آورده و نقطه برخورد را محاسبه کنید.

$$x-2+y-1=3 \Rightarrow y = -x+6$$

$$-x+2+y-1=3 \Rightarrow y = x+2$$

$$x-2-y+1=3 \Rightarrow y = x-4$$

$$-x+2-y+1=3 \Rightarrow y = -x$$

الف) $f(x) = x + \frac{1}{x} + 2\sqrt{x + \frac{1}{x}} \Rightarrow x + \frac{1}{x} > 0 \Rightarrow x > 0. R(x + \frac{1}{x}; x > 0) = [2, +\infty)$

$$\Rightarrow \sqrt{x + \frac{1}{x}} = [\sqrt{2}, +\infty) \Rightarrow 2\sqrt{x + \frac{1}{x}} = [2\sqrt{2}, +\infty) \quad x + \frac{1}{x} + 2\sqrt{x + \frac{1}{x}} \Rightarrow R_f = [2+2\sqrt{2}, +\infty)$$

ب) $y = (3 - \sin x)(1 + \sin x) = -\sin^2 x + 2\sin x + 3$

$$\xrightarrow{\max \sin x} \max \sin x = 1 \Rightarrow -\sin^2 x + 2\sin x + 3 = -1 + 2 + 3 = 4$$

$$\xrightarrow{\min \sin x} \min \sin x = -1 \Rightarrow -\sin^2 x + 2\sin x + 3 = -1 - 2 + 3 = 0$$

$$x = \frac{\pi}{2} \rightarrow \frac{\pi}{2} = \frac{\pi}{2} \Rightarrow -\sin^2 x + 2\sin x + 3 = -1 + 2 + 3 = 4$$

$$R_y = [0, 4]$$

$$f(x) = \frac{1}{c}x + 1$$

اندازه طبیعی

$$یک مجموعه = \{1, 2, \dots, 9\}$$

$$(1+2+\dots+9) = \frac{9 \times 10}{2} = 45$$

$$\Rightarrow \frac{1}{3}(1+2+\dots+9) + 1 \times 9 = \frac{1}{3}(45) + 9 = 15 + 9 = 24 \quad (\checkmark)$$

$$2 \text{ یا } 3, x=5 \Rightarrow \frac{1}{3}(5) + 1 = \frac{5}{3} + 1 = \frac{8}{3} \quad \frac{24}{3} \times 3 = 24 \times 3 = 72 \quad (\checkmark)$$

$$f(x) = \sqrt{ax^2 + bx + c}$$

فقط « بازه $[2, +\infty)$ » تعریف می شود $\Leftarrow$ خطی است بارش $2 \Leftarrow a=0$

$$\left. \begin{aligned} f(2) = \sqrt{4b+c} = 0 &\Rightarrow 4b+c=0 \\ f(3) = \sqrt{9b+c} = 1 &\Rightarrow 9b+c=1 \end{aligned} \right\} \Rightarrow \begin{aligned} b &= 1 \\ c &= -4 \end{aligned}$$

$$g(x) = \sqrt{x^2 + 4} \Rightarrow \mathbb{R} \setminus \{x^2 + 4\} \Rightarrow R = [2, +\infty)$$

$$\hookrightarrow R_{g(x)} = \sqrt{[4, +\infty)} = [2, +\infty)$$

$$f(x) = \frac{x^2 - 4x}{x+2} = \frac{x(x-4)}{x+2} = \frac{x(x+2)(x-2)}{x+2} = x^2 - 2x \quad ; \quad x \neq -2$$

$$x^2 - 2x \neq 1 \Rightarrow x^2 - 2x - 1 \neq 0 \Rightarrow (x-1)(x+2) \neq 0 \Rightarrow x \neq -2, x \neq 1$$

$$D_f = \mathbb{R} - \{-2, 1\}$$

$$R_f = [-1, +\infty) - \{1\} \Rightarrow C = -1 \Rightarrow 2C = -2 \Rightarrow f\left(\frac{a+b}{2}\right) = f\left(\frac{-2+1}{2}\right) = f\left(-\frac{1}{2}\right) = 1 - 2\left(-\frac{1}{2}\right) = 1 - (-1) = 2 \quad (\checkmark)$$

$$\hookrightarrow f(1) = 1 - 2 = -1$$

$$y = \sqrt[3]{x^3 + ax^2 + bx - 1} = \sqrt[3]{(x-K)^3}$$

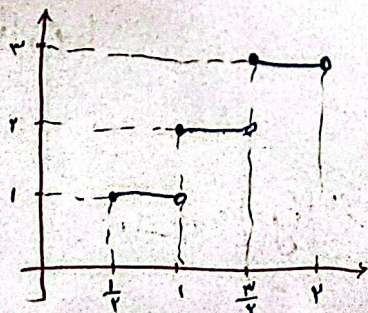
خطی است پس بصورت $\sqrt[3]{(x-K)^3}$ نوشته می شود

$$-1 = -K^3 \Rightarrow K^3 = 1 \Rightarrow K = 1$$

$$\sqrt[3]{(x-1)^3} = \sqrt[3]{x^3 - 3x^2 + 3x - 1} \quad \begin{cases} a = -3 \\ b = 3 \end{cases}$$

$$D_f = [-1, 1]$$

$$\sqrt[3]{(x-1)^3} = (x-1) \quad \begin{cases} x=1 & -3 \cdot 1 = -1 \\ x=0 & 3 \cdot 0 = 0 \end{cases} \Rightarrow R_f = [-1, 1]$$



$$y = \begin{cases} 0+1=1 & ; \quad \frac{1}{4} \leq x < 1 \\ 1+1=2 & ; \quad 1 \leq x < \frac{3}{4} \\ 2+1=3 & ; \quad \frac{3}{4} \leq x < 2 \end{cases}$$

در بازه $[1, 2]$