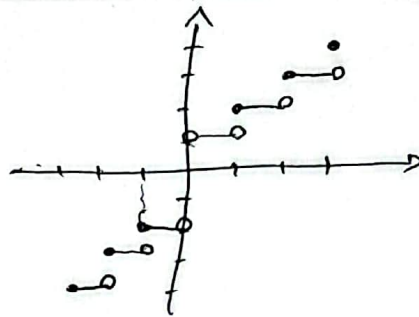


الف) $f(n) = 2 \sin^2 \left(n + \frac{\pi}{3} \right) - 1$ $\left\{ \begin{array}{l} 0 \leq \sin^2 t \leq 1 \Rightarrow -1 \leq 2 \sin^2 t - 1 \leq 1 \\ \Rightarrow -1 \leq f(n) \leq 1 \end{array} \right.$

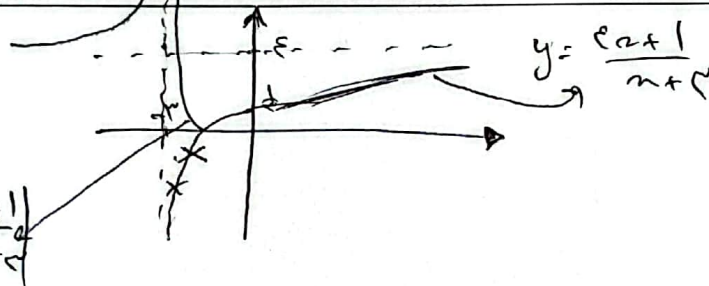
ب) $g(n) = 3 \cos^2 n - 4$ $-1 \leq \cos^2 n \leq 1 \Rightarrow 3 \cos^2 n - 4 \in [-1, -1]$

$y = \frac{n}{|n|} + [n]$

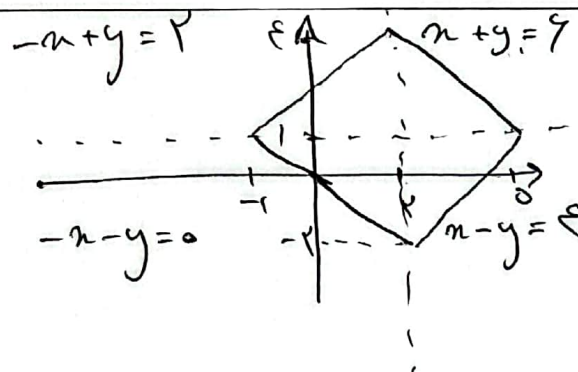
$y = \begin{cases} -4 & -4 \leq n < -2 \\ -3 & -2 \leq n < -1 \\ -2 & -1 \leq n < 0 \\ 1 & 0 \leq n < 1 \\ 2 & 1 \leq n < 2 \\ 3 & 2 \leq n < 3 \\ 4 & n = 3 \end{cases}$



$y = \left| \frac{\epsilon n + 1}{n + 2} \right|$



$|n - 2| + |y - 1| = 3$



الف) $f(n) = n + \frac{1}{n} + 2\sqrt{n + \frac{1}{n}}$ $\frac{b}{a} = -1$ $n + \frac{1}{n} \neq -1$ $n + \frac{1}{n} \geq 0 \Rightarrow n + \frac{1}{n} \geq 1$
 $\Rightarrow R_f: [1 + 2\sqrt{2}, +\infty)$

ب) $y = (3 - \sin n)(1 + \sin n) = -\sin^2 n + 2 \sin n + 3$

$\frac{b}{a} = \frac{2}{-1} = -2$ $\sin n = 1 \checkmark$
 $\sin n = 1 \Rightarrow -1 + 2 + 3 = 4$
 $\sin n = -1 \Rightarrow 1 - 2 + 3 = 2$ $\sin n = 0 \Rightarrow 3 \Rightarrow R_y: [2, 4]$

$$f(x) = -\frac{1}{x}x + 1 \quad D_f: \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

$$f(9) = 9 \times 1 + \left(\frac{-1}{9} \right) \left(\frac{9}{1} \right) = 9 - 1 = 8$$

$$f(x) = \sqrt{ax^2 + bx + c} \quad D_f: [1, +\infty) \quad f(1) = 1$$

$$a = 0 \quad b = 1 \quad c = 0 \quad \Rightarrow \frac{c}{b} \leq x \quad \frac{c}{b} = 1 \Rightarrow f(x) = \sqrt{bx - b}$$

$$f(1) = \sqrt{1 \cdot 1 - 1} = 0 \Rightarrow b = 1 \quad g(x) = \sqrt{x^2 - 1}$$

$$x^2 - 1 \geq 0 \Rightarrow R_g: [1, +\infty)$$

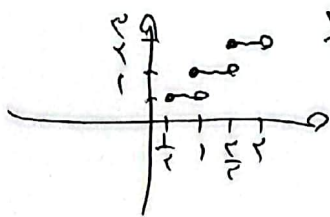
$$f(x) = \frac{x^2 - 1}{x + 1} \quad D_f: \mathbb{R} - \{-1\} \quad R_f: [1, +\infty) - \{1\}$$

$$f\left(\frac{a+b}{2}\right) = ? \quad \Rightarrow f(x) = \frac{x(x+1)(x-1)}{x+1} = x^2 - 1$$

$$f\left(\frac{1}{-1}\right) = f(-1) = 1 - 1 = 0$$

$$y = \sqrt{x^3 + ax^2 + bx - 1} \quad D_g: [-1, 1] \quad R_g: [-1, 1]$$

$$y = [x] + \left[x + \frac{1}{x}\right] \quad \frac{1}{x} < 1 \quad [x] = 0 \quad \left[x + \frac{1}{x}\right] = 1 \quad y = 1$$



$$y = 2$$