نام و نام خانوادگی ......[illegible]...... پاسخنامه تشریحی تکلیف شماره ...۱... کلاس ......[illegible]......

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الف) $y^3 - 3x^2 = 2x - 1 \Rightarrow y^3 = 3x^2 + 2x - 1 \Rightarrow y_1^3 = y_2^3 \Rightarrow y_1 = y_2 \Rightarrow$ تابع

ب) $x\cos y = y\cos x \Rightarrow x = \frac{\pi}{2} \Rightarrow \frac{\pi}{2}\cos y = 0 \Rightarrow y = k\pi + \frac{\pi}{2} \rightarrow$ تابع نیست

ج) $\frac{x+y}{2} = \sqrt{xy}$, $x.y > 0 \Rightarrow x+y = 2\sqrt{xy} \Rightarrow (\sqrt{x} - \sqrt{y})^2 = 0 \Rightarrow \sqrt{x} = \sqrt{y} \Rightarrow x = y$ $D_f \neq R$

تابع

$y^2 \neq -2$

د) $xy^2 + 2x - y^3 - 2y = 0 \Rightarrow x(y^2+2) - (y^3+2y) = x(y^2+2) - y(y^2+2) \Rightarrow (y^2+2)(x-y) = 0$ $x = y$

تابع

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$f(x) = \begin{cases} 2x^2 - b; & |x-1| \geq 2 \\ a + 2x; & -3 < x \leq -1 \end{cases}$ $\Rightarrow |x-1| \geq 2 \Rightarrow -2 \geq x - 1 \cancel{\geq} 2$

$\Rightarrow -1 \geq x \geq 3$

$\Rightarrow x = -1 \Rightarrow 2x^2 - b = a + 2x \Rightarrow 2 - b = a - 2 \Rightarrow a + b = 4$

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$f(x) = \sqrt{a - bx - x^2}$ $D_f = [a, b] \Rightarrow \sqrt{-(x^2 + bx - a)} \Rightarrow \sqrt{-(x-a)(x-b)}$

$ab = -a \Rightarrow b = -1$

$-(a+b) = b \Rightarrow a = -2b \Rightarrow b = -1, a = 2 \Rightarrow a + b = 2 - 1 = 1$

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$y = \frac{2}{\sqrt{ax^2 + bx + c}}$ $D_f = (-\infty, 2) \Rightarrow x < 2 \Rightarrow bx + c \xrightarrow[x=2]{} 2b + c = 0$

معادله نمی‌تواند درجه ۲ باشد $\leftarrow a = 0$

$\Rightarrow c = -2b$

$3a + c - 3|b| \Rightarrow 0 + (-2b) + 3b = b$

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الف) $f(x) = \sqrt[3]{\frac{1}{|x| - [x]}} \Rightarrow |x| - [x] \neq 0 \Rightarrow |x| \neq [x] \Rightarrow \begin{cases} x \in Z \Rightarrow |x| \neq x \\ \quad \rightarrow (-\infty, 0) \\ x \notin Z \Rightarrow |x| \neq x - 1 \end{cases}$

$\Rightarrow \begin{cases} x \neq -x + 1 \\ \cancel{x \neq x - 1} \end{cases} \rightarrow 2x \neq 1 \Rightarrow x \neq \frac{1}{2}$ $\rightarrow$ در جواب نیست

$D_f = R - W$

ب) $f(x) = \sqrt{x} + \sqrt{y} = 9$

$\rightarrow \sqrt{y} = 9 - \sqrt{x} \Rightarrow y = (9 - \sqrt{x})^2 \Rightarrow x \geq 0$, $x \leq 81$

$\Rightarrow D_f = [0, 81]$

$f(x)=\sqrt{\log_x^{3x-1}} \Rightarrow x>0, x\neq 1$ $\quad 3x-1>0 \Rightarrow x>\frac{1}{3}$ $\begin{cases} x>1 \Rightarrow 3x-1\geq 1 \Rightarrow x\geq \frac{2}{3} \\ 0<x<1 \Rightarrow 0<3x-1\leq 1 \Rightarrow \frac{1}{3}<x\leq \frac{2}{3} \end{cases} \Rightarrow D_f=(\frac{1}{3},\frac{2}{3}]\cup(1,+\infty)$ ۶

ب) $f(x)=\log \frac{1}{6+\sqrt{|x|}-|x|} \Rightarrow 6+\sqrt{|x|}-|x|>0 \Rightarrow -|x|+\sqrt{x}+6>0$

$\Rightarrow -(\sqrt{|x|}-3)(\sqrt{x}+2)>0 \Rightarrow \sqrt{x}<3 \Rightarrow |x|<9 \Rightarrow D_f=(-9,9)$

$y=\sqrt{\frac{3x+2}{x+b}+a}$ $\quad D_f=(2,+\infty) \Rightarrow \sqrt{\frac{3x+2+ax+ab}{x+b}}\geq 0 , x+b\neq 0 \Rightarrow x\neq -b$ ۷

$\sqrt{\frac{(3+a)x+ab+2}{x+b}} \Rightarrow$ چون دامنه ما باشد تابع خطی است

$\Rightarrow b+2=0 \Rightarrow b=-2$

$f(x)=\begin{cases} 2x-1 ; [x]>4 \\ 4x+3 ; [x]\leq 4 \end{cases}$ $\quad y=\sqrt{f(x)-x+1}$ $\quad \xrightarrow{I+II} D_f=[-\frac{4}{3},+\infty)$ ۸

$f(x)\begin{cases} 2x-1 ; x\geq 5 \Rightarrow y=\sqrt{2x-1-x+1}=\sqrt{x} \checkmark \quad I \\ 4x+3 ; x<5 \Rightarrow y=\sqrt{4x+3-x+1} \Rightarrow \sqrt{3x+4} \Rightarrow 3x+4\geq 0 \Rightarrow x\geq -\frac{4}{3} \quad II \end{cases}$

$y=\frac{24x^3+36x^2+18x+3}{(2x+1)^4}=\frac{3(8x^3+12x^2+6x+1)}{(2x+1)^4}=\frac{3\cancel{(2x+1)^3}}{\cancel{(2x+1)^4}}$ ۹

$=\frac{3}{2x+1} \Rightarrow D_f=\mathbb{R}-\{-\frac{1}{2}\}$

$y=\sqrt{1-\log(2x-a)}$ $\quad D_f=(b,c]$ ۱۰

$2x-a>0 , 2x-a\leq 10 \Rightarrow 0<2x-a\leq 10 \Rightarrow a<2x\leq 10+a$

$\Rightarrow \frac{a}{2}<x\leq \frac{10+a}{2} \Rightarrow b=\frac{a}{2}, c=\frac{10+a}{2} \Rightarrow c-b=\frac{10}{2}=5$