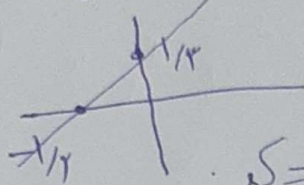
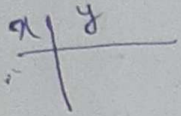


$$2y = 2x - 1 \rightarrow 2y + 1 = 2x \rightarrow x = \frac{2}{2}y + \frac{1}{2} \Rightarrow \frac{2}{2}y + \frac{1}{2} = \frac{2}{2}x + \frac{1}{2}$$



$$K = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{12}$$

۱

الف - در شرط $\frac{a+b}{2} = \frac{b}{2a} = 1$ در شرط اول داریم $a = 2$ و در شرط دوم داریم $b = 2$

$$y = \sqrt{x-2} + 1 - f^{-1}$$

در شرط دوم داریم $b = 2$

۲

$$D_{f^{-1}} = [4, +\infty) = R_f$$

ف. چون در شرط اول داریم $a = 2$

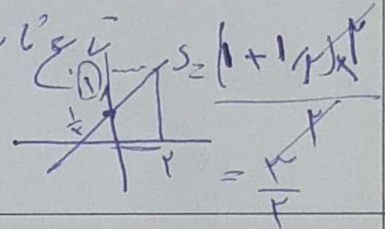
در شرط دوم داریم $b = 2$

$$f(x) = 2x - \sqrt{4 - 2x(2-x)} = 2x - \sqrt{4(2-x)} = 2x - 2\sqrt{2-x}$$

$$\rightarrow x < -2 \rightarrow f(x) = 2x - 2$$

$$x > 2 \rightarrow f(x) = 2$$

$$y = 2x - 2 \rightarrow f^{-1} = \frac{1}{2}y + 1$$



۳

۴

حداکثر مقدار K زمانی که f و f^{-1} در $[a, b]$ باشند

$$K - \varepsilon = 2K + 1$$

$$\rightarrow K = -2 \rightarrow K = (-\infty, -2]$$

۵

$$f(x) = r|x+1| \begin{cases} \varepsilon x & x \geq 0 \\ rx & x < 0 \end{cases}$$

$$\rightarrow f^{-1} = \begin{cases} \frac{1}{\varepsilon} x \\ \frac{1}{r} x \end{cases}$$

$$x \geq 0 \rightarrow f^{-1}(x) = \frac{r}{\varepsilon} x - \frac{1}{\varepsilon} |x|$$

$$x < 0 \rightarrow f^{-1}(x) = \frac{r}{\varepsilon} x - \frac{1}{\varepsilon} |x|$$

$$\rightarrow a = \frac{r}{\varepsilon} \quad b = -\frac{1}{\varepsilon}$$

6

$$f(x) = \frac{10}{x+1} = -10 \rightarrow M = -1$$

$$f^{-1}(x) = \frac{rx + \varepsilon}{x - r} \quad a = d \cdot \frac{1}{r}$$

$$\frac{rx + \varepsilon}{x - r} + x = \frac{ax + 1 + \varepsilon}{x - r} = x \rightarrow rx + \varepsilon = x^2 - rx \rightarrow rx = -\varepsilon$$

$$x = -\varepsilon/r$$

7

$$g^{-1}(\varepsilon) = a \rightarrow g(a) = \varepsilon \rightarrow \frac{1}{r} = a \rightarrow x = ra \rightarrow rx = \varepsilon a$$

$$f(r - \varepsilon a) = r - g(a) = r - \varepsilon = -r$$

$$\rightarrow f(r - \varepsilon a) = -r \rightarrow f^{-1}(-r) = r - \varepsilon a$$

$$\rightarrow ra + r - \varepsilon a = r$$

8

$$y = \frac{\omega x - 13}{1 - x} \xrightarrow{\text{cross multiply}} y = \frac{\omega x - 13}{1 + x} \Rightarrow$$

$$\rightarrow y = \frac{\omega x + r}{x - 1} = \frac{\omega x - 13}{1 - x} \rightarrow (\omega x + r)(1 - x) = (x - 1)(\omega x - 13)$$

$$\rightarrow 10x^2 - 10x + 10 = 0 \rightarrow \frac{-b}{a} = S = \frac{r_0}{10} = r$$

9

$$y = x + \varepsilon[n] \quad [y] = \omega[n] \quad [n] = \frac{[y]}{\omega}$$

$$f^{-1}([y]) = \left[\frac{n}{\omega} \right] \quad x + \varepsilon[n] = \left[\frac{n}{\omega} \right] = \varepsilon/1[n]$$

10

$$f(x) = r|x+1| \begin{cases} \varepsilon x & x \geq 0 \\ rx & x < 0 \end{cases}$$

$$\rightarrow f^{-1} = \begin{cases} \frac{1}{\varepsilon} x \\ \frac{1}{r} x \end{cases}$$

$$x \geq 0 \rightarrow f^{-1}(x) = \frac{r}{\lambda} x - \frac{1}{\lambda} |x|$$

$$x < 0 \rightarrow f^{-1}(x) = \frac{r}{\lambda} x - \frac{1}{\lambda} |x|$$

$$\rightarrow a = \frac{r}{\lambda} \quad b = -\frac{1}{\lambda}$$

$$f(x) = \frac{10}{x+1} = -10 \rightarrow M = -r$$

$$f^{-1}(x) = \frac{rx + \varepsilon}{x - r}$$

$$a = d \cdot \frac{1}{r}$$

$$\frac{ra + \varepsilon}{x - r} + x = \frac{ax + a + \varepsilon}{x - r} = x \rightarrow ax + a + \varepsilon = x^2 - rx \rightarrow rx = -\varepsilon$$

$$x = -\varepsilon/r$$

$$g^{-1}(\varepsilon) = a \rightarrow g(a) = \varepsilon \rightarrow \frac{a}{r} = a \rightarrow a = ra \rightarrow ra = \varepsilon a$$

$$f(r - \varepsilon a) = r - g(a) = r - \varepsilon = -r$$

$$\rightarrow f(r - \varepsilon a) = -r \rightarrow f^{-1}(-r) = r - \varepsilon a$$

$$\rightarrow ra + r - \varepsilon a = r$$

$$y = \frac{\omega x - 1r}{1 - x} \xrightarrow{\text{cross multiply}} y = \frac{\omega x - 1r}{1 + x} \Rightarrow$$

$$\rightarrow y = \frac{\omega x + r}{x - 1} = \frac{\omega x - 1r}{1 - x} \rightarrow (\omega x + r)(1 - x) = (x - 1)(\omega x - 1r)$$

$$\rightarrow 10x^2 - 10x + 10 = 0 \rightarrow \frac{-b}{a} = S = \frac{r_0}{10} = r$$

$$y = x + \varepsilon[x] \quad [y] = \omega[x] \quad [x] = \frac{[y]}{\omega}$$

$$f^{-1} \quad [y] = \left[\frac{x}{\omega} \right] \quad x + \varepsilon[x] = \left[\frac{x}{\omega} \right] = \varepsilon / \omega [x]$$