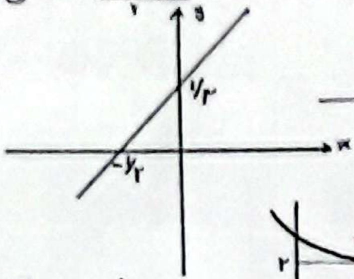
محمد صالح محسنی دوازدهم پسر B تکلیف شماره ۱

①

$2y = 3x - 1 \longrightarrow y = \frac{3x-1}{2} \xrightarrow{\text{وارون}} x = \frac{3y-1}{2} \Rightarrow \frac{2x+1}{3} = y$

$\frac{2}{3}x + 1/3 = y$

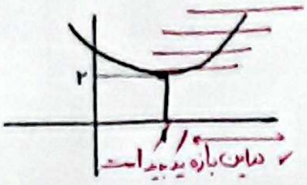


$\longrightarrow S = 1/2 \times 1/2 \times 1/3 = \frac{1}{12}$

②

A) $x^2 - 2x + 3 = \,,\ x \in [1, +\infty)$

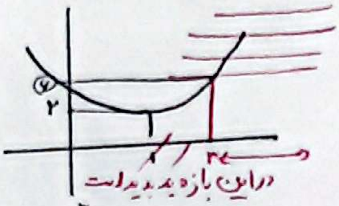
$R_f\ [2, +\infty)$



$\longrightarrow x^2 - 2x + 1 + 2 = (x-1)^2 + 2 = y \longrightarrow (y-1)^2 + 2 = x \longrightarrow \sqrt{x-2} + 1 = y \quad x \in [2, +\infty)$

B) $x^2 - 2x + 3 = \,,\ x \in [3, +\infty)$

$9 - 6 + 3 = ⑥ \qquad R_f\ [6, +\infty)$



$x^2 - 2x + 3 = (x-1)^2 + 2 = y \longrightarrow \sqrt{y-\ }\ +\ = x \longrightarrow \sqrt{x-2} + 1 = y \quad x \in [6, +\infty)$

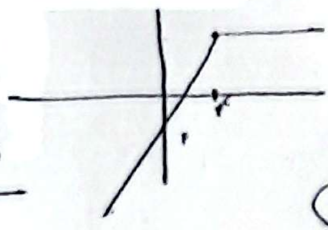
③

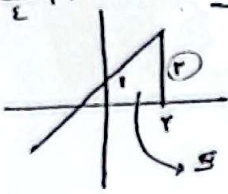
$\frac{x - \sqrt{16 - 4x(4-x)}}{2} \longrightarrow 2x - |2(x-2)| \longrightarrow x > 2 \quad 4$

$\longrightarrow x < 2 \quad 4x - 4$

$16 - 16x + 4x^2 \longrightarrow$

$4x^2 - 16x + 16 \longrightarrow 4(x-2)^2 \longrightarrow f^{-1} = \frac{x}{4} + 1$





$\rightarrow S = \frac{2 \times 4}{2} = ④$

④

$f(x_1) = f(x_2) \Rightarrow \frac{x_1}{\sqrt{x_1^2 - 5}} = \frac{x_2}{\sqrt{x_2^2 - 5}}$

$\frac{x_1^2}{x_1^2 - 5} = \frac{x_2^2}{x_2^2 - 5} \Rightarrow x_1^2 x_2^2 - 5x_2^2 = x_1^2 x_2^2 - 5x_1^2 \longrightarrow x_1^2 = x_2^2 \rightarrow x_1 = \pm x_2$

پس یک به یک نمی‌تواند ۲ x متفاوت داشته باشد پس $x_1 = x_2$ است پس تابع یک به یک است

$\frac{x_1}{\sqrt{x^2 - 5}} = y \Rightarrow x = \frac{y}{\sqrt{y^2 - 5}} = x^2 = \frac{y^2}{y^2 - 5} \Rightarrow x^2 y^2 - 5x^2 = y^2$

$y^2 = \frac{5x^2}{x^2 - 1} \Rightarrow y = \sqrt{\frac{5x^2}{x^2 - 1}} = \frac{\sqrt{5}\, x}{\sqrt{x^2 - 1}}$

۵-

$f(x) = x^2 - 4x + k$ $\quad x \geq 2$ ⟵ تابع صعودی

$g(x) = -x^2 + ax + 2k$ $\quad x < 2$ ⟵ تابع نزولی

$f(2) \geq g(2) \longrightarrow -4 + k \geq 8 + 4k \longrightarrow -12 \geq 3k \longrightarrow -4 \geq k$ ✓

۶-

$2x + |x| \longrightarrow \begin{cases} 3x & , x \geq 0 \longrightarrow \frac{3}{2}x \quad x \geq 0 \\ x & , x < 0 \longrightarrow \frac{1}{2}x \quad x < 0 \end{cases}$

$ax + b|x| \longrightarrow \begin{cases} (a+b)x & x \geq 0 \longrightarrow a+b = \frac{1}{4} \\ (a-b)x & x < 0 \longrightarrow a-b = \frac{1}{2} \end{cases}$

$\longrightarrow 2a = \frac{3}{4} \rightarrow a = \frac{3}{8}$, $b = -\frac{1}{8}$

$\longrightarrow ab = \frac{3}{8} \times -\frac{1}{8} = -\frac{3}{64}$

۷

$\frac{3x+2}{x+m} \longrightarrow \frac{9+2}{3+m} = -10 \longrightarrow m = -4$

$f_0 f^{-1}(x) = x$ $\quad x + \frac{3x+4}{x-2} = x \Rightarrow \frac{3x+4}{x-2} = 0$

$\frac{3x+4}{x-2}$ $\quad x = \frac{-4}{3}$

۸

$f(4-2x) = 2 - g(\frac{x}{2})$

$2g^{-1}(2) + f^{-1}(10)$

$f(4-2x) = -2 = 2 - g(\frac{x}{2}) \longrightarrow g(\frac{x}{2}) = 4 \longrightarrow g^{-1}(4) = \frac{1}{2}x$

$\frac{2g^{-1}(4)}{2x} + \frac{f^{-1}(10)}{6-2x} = ?$

۹

$-x \rightarrow \frac{-5x-12}{1+x}$ $\xrightarrow{-y}$ $\frac{5x+12}{x+3}$ $\xrightarrow{x=x-2}$ $\frac{5x+2}{x-1}$ $\xrightarrow{y-2}$ $\frac{5x+2}{x-1} - 2$

$\longrightarrow \frac{3x+4}{x-1} = f(x)$ $\quad \frac{4x+4}{x-1} = x \longrightarrow \frac{-x^2+2x+4}{x-1}$

$-x^2 + 3x + 4 = 0 \longrightarrow S = \frac{-b}{a} = 3$ ۱۰

$$f(x) = x + \lfloor x \rfloor$$

$$= x = y + \lfloor y \rfloor \longrightarrow y = x - \lfloor y \rfloor$$

$$\lfloor x \rfloor = \lfloor y + \lfloor y \rfloor \rfloor = \lfloor x \rfloor = 2\lfloor y \rfloor \Rightarrow \lfloor y \rfloor = \frac{\lfloor x \rfloor}{2}$$

$$y = x - \frac{\lfloor x \rfloor}{2} = g(x)$$

$$x + \lfloor x \rfloor - x + \frac{\lfloor x \rfloor}{2} = \frac{3\lfloor x \rfloor}{2} = f_{-g}(x)$$