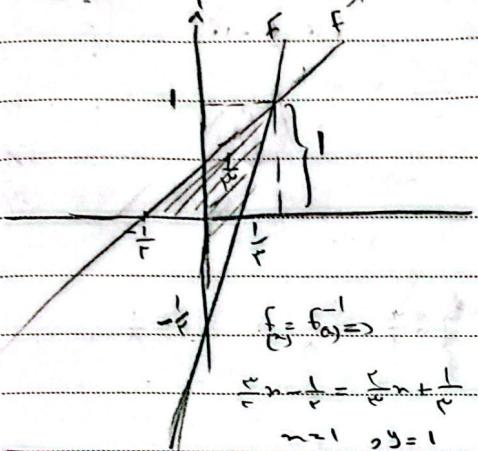


میرزا خدا

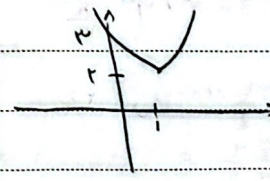
کیوان مکارم سب

تلف سوار ۱۰
 $y = \frac{1}{x}x - 1 \xrightarrow{f^{-1}} x = \frac{1}{y}y - 1 \rightarrow \frac{1}{y}y = x + 1 \rightarrow \frac{1}{y} = x + 1 \rightarrow y = \frac{1}{x+1} = f^{-1}$



$S = \frac{\text{ارتفاع} \times \text{عرض}}{2} = \frac{1 \times 5}{2} = \frac{5}{2}$ ✓

(۲) $y = x^2 - 2x + 3 \xrightarrow{x = \frac{-b}{2a}} x_{\text{min}} = \frac{2}{2} = 1$



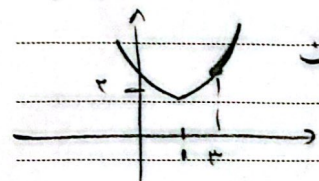
با توجه به شکل تابع از

۱ به بعد، یک به یک است. ✓

$\rightarrow y = x^2 - 2x + 1 + 2 \rightarrow y = (x-1)^2 + 2$

$\Rightarrow x = (y-2)^{\frac{1}{2}} + 1 \rightarrow (y-2)^{\frac{1}{2}} = x-1 \rightarrow f^{-1}(y) = \sqrt{y-2} + 1$ ✓

۱- $y = x^2 - 2x + 3 \xrightarrow{x = \frac{-b}{2a}} x_{\text{min}} = \frac{2}{2} = 1 \rightarrow y_{\text{min}} = 1 - 2 + 3 = 2$



طبق بخش الف

با توجه به شکل، از ۲ به بعد تابع یک به یک است. ✓

$\rightarrow f^{-1}(y) = \sqrt{y-2} + 1$ ✓

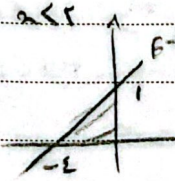
(۱۰) $f(x) = 2x - \sqrt{x^2 - 4x + 4} \Rightarrow 2x - \sqrt{x^2 - 4x + 4} = f(x)$

$\Rightarrow f(x) = 2x - 2\sqrt{x^2 - 4x + 4} \rightarrow f(x) = 2x - 2\sqrt{(x-2)^2} \xrightarrow{x \geq 2} f(x) = 2x - 2(x-2) = 2x - 2x + 4 = 4$

$= 4 \rightarrow$ تابع یک به یک نیست. $\xrightarrow{x < 2} f(x) = 2x - 2(2-x) = 2x - 4 + 2x = 4x - 4$

با توجه به اینکه $f(x) = 4x - 4$ است. $\rightarrow f(x) = 4x - 4$

(۰، ۲]



$\rightarrow S = \frac{1 \times 4}{2} = 2$ ✓

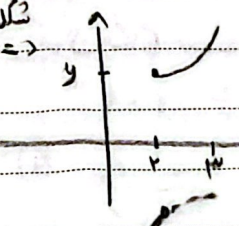
$\rightarrow x = 4y - 4 \rightarrow f^{-1}(y) = \frac{1}{4}y + 1$

(۴) $y = \frac{x_1}{\sqrt{x_1^2 - a}} = \frac{x_2}{\sqrt{x_2^2 - a}} \xrightarrow{x_1^2 = a} \frac{x_1^2}{x_1^2 - a} = \frac{x_2^2}{x_2^2 - a} \Rightarrow \frac{x_1^2}{x_1^2 - a} - \frac{x_2^2}{x_2^2 - a} = 0$

$\Rightarrow x_1^2 = a \rightarrow x_1 = \pm \sqrt{a}$ $\rightarrow$ تابع یک به یک نیست.

$x = \frac{y}{\sqrt{y^2 - a}} \Rightarrow x^2 = \frac{y^2}{y^2 - a} \rightarrow x^2 y^2 - a x^2 = y^2 \rightarrow x^2 y^2 - y^2 = a x^2$

$\Rightarrow y^2 = \frac{a x^2}{x^2 - 1} \rightarrow f^{-1}(y) = \frac{\sqrt{a x}}{\sqrt{x^2 - 1}}$ ✓ (تابع یک به یک نیست)

$x > 2 \rightarrow \text{اولی } m = \frac{\epsilon}{2} = 2 \rightarrow x < 2 \rightarrow \text{اولی } m = \frac{4}{2} = 2$ (a)
 $\Rightarrow$  $\Rightarrow$ $-x^2 + 4(2) + 3K \leq 2^2 - \epsilon(2) + K$
 $\Rightarrow$ $-x^2 + 4(2) + 3K \leq 4 - \epsilon(2) + K$
 $\Rightarrow$ $-x^2 + 4(2) + 3K \leq 4 - \epsilon(2) + K$
 $\Rightarrow$ $2K \leq -1\epsilon \rightarrow \boxed{K \leq -\frac{1}{2}\epsilon}$

$f(x) = \begin{cases} \epsilon x & x > 0 \\ 2x & x < 0 \end{cases} \rightarrow g(x) = \begin{cases} \frac{x}{\epsilon} & x > 0 \\ \frac{x}{2} & x < 0 \end{cases}$

$\begin{cases} a + b = \frac{1}{\epsilon} \\ a - b = \frac{1}{\epsilon} \end{cases} \Rightarrow \begin{cases} a + b = \frac{1}{\epsilon} \\ a - b = \frac{1}{\epsilon} \end{cases} \rightarrow 2a = \frac{2}{\epsilon} \rightarrow a = \frac{1}{\epsilon}, b = -\frac{1}{\epsilon}$
 $\Rightarrow ab = \frac{1}{\epsilon} \times (-\frac{1}{\epsilon}) = -\frac{1}{\epsilon^2}$

$f(x) = -b \rightarrow \frac{4 + \epsilon}{2 + m} = -10 \rightarrow b = -10 - 10m \rightarrow m \rightarrow 0$ (v)

$\Rightarrow f(x) = \frac{4n + \epsilon}{2 - n} \rightarrow f(f(m)) = \frac{4(\frac{4n + \epsilon}{2 - n}) + \epsilon}{2 - \frac{4n + \epsilon}{2 - n}}$
 $\Rightarrow \frac{4n + \epsilon}{2 - n} = \frac{14n}{14} = n = f \circ f(m)$

$y = \frac{4n + \epsilon}{2 - n} \Rightarrow xy - 4y = 4n + \epsilon \rightarrow xy - 4n = 4y + \epsilon$
 $\Rightarrow x(y - 4) = 4y + \epsilon \rightarrow x = \frac{4y + \epsilon}{y - 4} \rightarrow f^{-1}(y) = \frac{4y + \epsilon}{y - 4}$
 $\Rightarrow x + \frac{4n + \epsilon}{2 - n} = x \rightarrow \boxed{x = -\frac{\epsilon}{4}}$

$f(4m) = 4 - g(\frac{x}{4}) \rightarrow f^{-1}(4) = 4 - 4m \rightarrow f(4 - 4m) = 4$ (A)
 $\rightarrow \frac{x}{4} \xrightarrow{g} 4 \rightarrow g^{-1}(4) = \frac{x}{4} \rightarrow 4g^{-1}(4) = x$
 $\rightarrow \epsilon g^{-1}(4) = 4x \rightarrow \epsilon g^{-1}(4) = f^{-1}(4 - \epsilon)$

$\Rightarrow 4m + 4 - 4m = 4 \checkmark$

فرضه شو
 $-y = \frac{-2n - 12}{n + 1} \rightarrow y = \frac{2n + 12}{n + 1} \xrightarrow{\text{تقسیم بر } n+1} y = \frac{2(n-1) + 14}{(n-1) + 1} \quad (9)$

داده های ما
 $f = \frac{2n + 12}{n - 1} - 12 = \frac{2n + 12 - 12n + 12}{n - 1} = \frac{-10n + 24}{n - 1} = f(n)$

$y = \frac{2n + 4}{n - 1} \xrightarrow{f^{-1}(n)} x = \frac{2y + 4}{y - 1} \Rightarrow 2y - 2 = 2y + 4 \Rightarrow 2y - 2y = 2 + 4$

$\Rightarrow y(n - 2) = 2 + 4 \rightarrow f^{-1}(n) = \frac{2 + 4}{n - 2} \rightarrow \frac{2 + 4}{n - 2} = \frac{2(n + 4)}{n - 1}$

$\Rightarrow n^2 + 2n - 4 = 2(n^2 + n - 4) \Rightarrow n^2 + 2n - 4 = 2n^2 + 2n - 8$

$\Rightarrow n^2 - 2n - 4 = 0 \rightarrow \Delta = 2^2 - 4 \cdot 1 \cdot (-4) = 4 + 16 = 20 \rightarrow \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-2 \pm \sqrt{20}}{2} = \frac{-2 \pm 2\sqrt{5}}{2} = -1 \pm \sqrt{5} \quad (10) \checkmark$

$y = n + \varepsilon[n] \xrightarrow{[n]=K} y = n + \varepsilon K \xrightarrow{\text{تقسیم بر } n} n = y + \varepsilon K \quad (10)$

$\rightarrow g(n) = n - \varepsilon K \rightarrow g(n) = n - \varepsilon[n] \rightarrow f(n) - g(n) \rightarrow$

$= n + \varepsilon[n] - n - \varepsilon[n] = 0 \quad \checkmark$