نام و نام خانوادگی ....[illegible].... پاسخنامه تشریحی تکلیف شماره ..۵.. کلاس ....[illegible]....

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$2y = 3x - 1 \Rightarrow y = \frac{3}{2}x - \frac{1}{2} \Rightarrow f(x) = \frac{3}{2}x - \frac{1}{2}$

$\Rightarrow f^{-1}(x) = \frac{2x+1}{3} \Rightarrow$

$\Rightarrow S_{ABC} = \frac{1}{3} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{12}$

$\frac{2x+1}{3} = 0 \Rightarrow x = -\frac{1}{2}$

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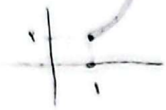
الف) $x^2 - 2x + 3 \Rightarrow D_f = [1, +\infty)$

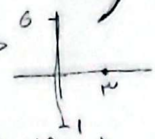


$\Rightarrow (x-1)^2 + 2$

$\Rightarrow y = (x-1)^2 + 2 \Rightarrow \sqrt{y-2} + 1 = x$

$\Rightarrow f^{-1}(x) = \sqrt{x-2} + 1 \Rightarrow D_{f^{-1}} = R_f = [2, +\infty)$

ب) $x^2 - 2x + 3 \quad x \in [3, +\infty)$



$\Rightarrow$ ↳ $(x-1)^2 + 2$

$\Rightarrow f^{-1}(x) = \sqrt{x-2} + 1$

$\Rightarrow D_{f^{-1}} = R_f = [6, +\infty)$

$R_{f^{-1}} = D_f = [3, +\infty)$

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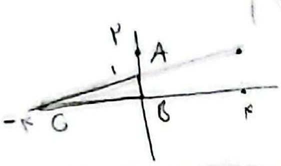
$f(x) = 2x - \sqrt{16 - 4x(4-x)} \Rightarrow 2x - \sqrt{4x^2 - 16x + 16} = 2x - 2|x-2|$

$2x - 2|x-2| \begin{cases} 2x - 2x + 4 & x \geq 2 \\ 2x + 2x - 4 & x < 2 \end{cases} \Rightarrow 4 ; x \geq 2 \Rightarrow f(x) = 4x - 4$

$\boxed{4x - 4 ; x < 2} \Rightarrow f^{-1}(x) = \frac{x}{4} + 1$

$R_{f^{-1}} = D_f = (-\infty, 2)$

$D_{f^{-1}} = R_f = (-\infty, 4)$



$S_{ABC} = \frac{4 \times 1}{2} = 2$

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$y = \frac{x}{\sqrt{x^2 - 5}} \rightarrow \frac{x_1}{\sqrt{x_1^2 - 5}} = \frac{x_2}{\sqrt{x_2^2 - 5}} \overset{\star}{\Rightarrow} \frac{x_1^2}{x_1^2 - 5} = \frac{x_2^2}{x_2^2 - 5} \Rightarrow x_1^2 x_2^2 - 5x_1^2 = x_1^2 x_2^2 - 5x_2^2$

$\Rightarrow x_1^2 = x_2^2 \Rightarrow x_1 = \pm x_2$

← از آنجایی که اگر $x_1 = -x_2$ باشد آنگاه آنها مختلف العلامت می‌شوند $x_1 = -x_2$ جواب نمی‌باشد و تنها $\boxed{x_1 = x_2}$ جواب است پس تابع یک به یک است

$x = \frac{y}{\sqrt{y^2 - 5}} \Rightarrow x^2 = \frac{y^2}{y^2 - 5} \Rightarrow y^2 = \frac{5x^2}{x^2 - 1} \Rightarrow y = \sqrt{\frac{5x^2}{x^2 - 1}}$

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$g(x) \begin{cases} x^2 - 4x + k & ; x \geq 2 \\ x^2 + 4x + 3k & ; x < 2 \end{cases} \Rightarrow$

$f(2) = 4 - 8 + k = k - 4$

$f(2) = -4 + 12 + 3k = 3k + 8$

$\Rightarrow 3k + 8 \leq k - 4 \Rightarrow 2k \leq -12 \Rightarrow \boxed{k \leq -6}$

$f(x)=3x+|x| \Rightarrow \begin{cases} 3x+x & x\geq 0 \\ 3x-x & x<0 \end{cases} \Rightarrow 4x=y \Rightarrow f^{-1}(x)=\begin{cases} \frac{x}{4} & x\geq 0 \\ \frac{x}{2} & x<0 \end{cases}$ ۶

$\Rightarrow g(x)=ax+b|x|=\begin{cases} \frac{2x}{8} ; x\geq 0 \\ \frac{4x}{8} ; x<0 \end{cases} \Rightarrow \begin{cases} \frac{3x}{8}-\frac{x}{8} \\ \frac{3x}{8}+\frac{x}{8} \end{cases} \Rightarrow \frac{3}{8}(x)-\frac{1}{8}|x|$

$\Rightarrow a=\frac{3}{8}, b=\frac{-1}{8} \Longrightarrow ab=-3/64$

$f(x)=\frac{3x+4}{x+m} \Rightarrow \Big|^{2}_{-10} \Rightarrow \frac{6+4}{2+m}=-10 \Rightarrow \boxed{m=-3} \Rightarrow f(x)=\frac{3x+4}{x-3}$ ۷

$\Rightarrow f^{-1}(x)=\frac{3x+4}{x-3} \Rightarrow f(x)=f^{-1}(x) \quad \Big| \Rightarrow f\circ f^{-1}(x)+f^{-1}(x)$

$x+\frac{3x+4}{x-3}=x \Rightarrow \frac{3x+4}{x-3}=0 \Rightarrow 3x+4=0 \Rightarrow \boxed{x=-\frac{4}{3}}$

$f(3-2x)=2-g\left(\frac{x}{2}\right)$ ۸

$f^{-1}(-2)+4g^{-1}(4) \Rightarrow f(3-2x)=-2 \Rightarrow 2-g\left(\frac{x}{2}\right)=-2 \Rightarrow g\left(\frac{x}{2}\right)=4$

$\Rightarrow g^{-1}(4)=\frac{x}{2}, \quad f^{-1}(-2)=3-2x$

$\Rightarrow 3-2x+2x=3$

$y=\frac{5x-13}{1-x} \Rightarrow y=-\frac{-5x-13}{1+x} \Rightarrow y=\frac{5x+13}{1+x} \Rightarrow \frac{5(x-2)+13}{x-2+1}-3$

$\Rightarrow \frac{5x+3}{x-1}-3 \Rightarrow \frac{5x+3-3x+3}{x-1}=\frac{2x+6}{x-1} \Rightarrow f(x)=\frac{2x+6}{x-1} \Rightarrow f^{-1}(x)=\frac{x+6}{x-2}$

$\Rightarrow \frac{x+6}{x-2}=\frac{2x+6}{x-1} \Rightarrow x^2+6x-x-6=2x^2+6x-4x-12 \Rightarrow x^2-3x-6 \Rightarrow S=3$

$f(x)=x+4[x] \quad g(x)=f^{-1}(x) \Rightarrow x=4[y]+y \Rightarrow [x]=5[y] \Rightarrow [y]=\frac{[x]}{5}$

$f-g(x) \Rightarrow f-f^{-1}(x) \Rightarrow y=x-4[y] \Rightarrow y=x-\frac{4[x]}{5}$

$\Rightarrow x+4[x]-x+\frac{4[x]}{5} \Rightarrow \boxed{\frac{24[x]}{5}}$