پرهام یراقی　　　　　۱۸٫۲۵　　　　/ /

۱)
$f(x)=2y$　　$f^{-1}=3y$
$f(x)=4x-1$　　$\frac{\frac{1}{3}+\frac{1}{4}}{2}=\frac{1}{12}$　　$f^{-1}=\frac{3x+1}{3}-1=\frac{2}{3}x+\frac{1}{3}$

۲

۲) الف)
$y=x^2-2x+3$ و $x\in[3,+\infty)$
$\frac{-b}{2a}=1$ ✓ یک به یک　　$f^{-1}: x-(y-1)^2=2$
$\to y-1=\sqrt{x-2} \to y=\sqrt{x-2}+1$　　۲
$Df^{-1}\in[2,+\infty)$

ب)
$y=x^2+2x+3 \to x\in[3,+\infty)$　　$\frac{-b}{2a}=1$　　یک به یک
$y^{-1}=\sqrt{x-2}+1$ غلط　　$Rf^{-1}\in[6,+\infty)$

۳) ۱٫۲۵
$f(x)=2x-\sqrt{16-4x(4-x)}=2x-\sqrt{4(4-x^2)}=2x-2|4x|$
$Df\in(-\infty,4]$
$x\in(-\infty,4]$
$f(x)=4x-\cancel{8}^{4}\Rightarrow f(x)^{-1}=\frac{1}{4}x+2$　　$Rf=(-\infty,8]=Df^{-1}_{(x)}$　　$\frac{48}{2}=24$

$f(n)=4n-4 \to f^{-1}(n)=\frac{n+4}{4}$, $Rf=(-\infty,4]$

$S=\frac{4\times 4}{2}=8$

۴)
$y=\frac{x}{\sqrt{x^2-5}}$　　$\frac{x_1}{\sqrt{x_1^2-5}}=\frac{x_2}{\sqrt{x_2^2-5}}\Rightarrow x_1^2,\ x_2^2-5x_1^2-5x_1^2=x_1^2x_2^2\to x_1^2=x_2^2$
$x_1=\pm x_2 \Rightarrow x_1=x_2$　　یک به یک　　$f^{-1}\to x=\frac{y}{\sqrt{y^2-5}}\Rightarrow y^2=\frac{5x^2}{x^2-1}$　　۲
$y=\pm\frac{\sqrt{5x^2}}{\sqrt{x^2-1}}$

۵)
$g(x)$　x^2-4x+4　　$x\geq 2$　　$\frac{-b}{2a}=2$
$x^2+4x+3x$　　$x<2$　　$\frac{-b}{2a}=2$
$4\geq 3k+8\Rightarrow k\leq 6$　　۲

(6

$f(x) = 3x + |x| \begin{cases} 4x & x \geq 0 \\ 2x & x < 0 \end{cases}$ $\quad f^{-1}(x) \begin{cases} \frac{1}{4}x & x \geq 0 \\ \frac{1}{2}x & x < 0 \end{cases}$

2

$f^{-1}(x) = \frac{3x}{8} - \frac{|x|}{8} = \frac{15x}{40} - \frac{5}{40}|x| \Rightarrow a = \frac{3}{8},\ b = -\frac{1}{8} \quad ab = -\frac{3}{64}$

(7 $\quad$ 3

$f(x) = \frac{3x+4}{x+m} \quad f(x) = -10 \quad 10m - 30 = \Rightarrow m = -3$

$\Rightarrow x \neq 3 ,\ f(-10) = 3 \to f^{-1} = f \quad y = f \circ f^{-1}(x) = f^{-1}(x) = \frac{3x+4}{x-3}$

$= x \Rightarrow \frac{3x+4}{x-3} = 0 \quad 3x + 4 = x = -\frac{4}{3}$

(8 $\quad$ 2

$f(3-2x) = 2 - g(\frac{x}{2}) \Rightarrow g(\frac{x}{2}) = 2 - f(1-2x) \to g(x) = 4$

$f = 2 - f(3-4x) \quad f^{-1}(2-t) = 3 - 10t \to f^{-1}(2-t) = 3 - 4g(1)^{-1} = 3 - f(-2)$

$g^{-1}(1) = x \to 4g^{-1}(4) = f^{-1}(-2) = 3$

(9

$g = \frac{5x-13}{1-x} \xrightarrow{\text{منفی}} \frac{5x+3}{x-1} \to \frac{5x+3}{x-1} - 3 = \frac{5x+3-3x+3}{x-1}$

$= \frac{2x+6}{x-1} \Rightarrow f(x) = \frac{2x+6}{x+1} \Rightarrow f^{-1}(x) = y = \frac{2y+6}{y-1} \Rightarrow y = \frac{6+x}{x-2} \Rightarrow$ 2

$f^{-1}(x) \Rightarrow \frac{6+x}{x-2} = \frac{2x+1}{x-1} \Rightarrow x^2 - 5x - 6 \Rightarrow (x-6)(x+1) \Rightarrow 2x^2 - 2x - 12 \to$

$x^2 - 3x - 6 = 0 \to \frac{-b}{a} = -\frac{(-3)}{1} = 3$

(10 $\quad$ 1

$f(x) = x + 4[x] \Rightarrow g(x) = f^{-1}(x) \Rightarrow x - y + 4[y] \Rightarrow y = x - 4[x] = g(x)$

$[y] = [x + 4[x]] \Rightarrow [y] = 5[x] \Rightarrow f(x) - g(x) = x + 4[x] - x$

$+ 20[x] = 24[x]$ $\quad [y] = 5[x] \to [x] = \frac{[y]}{5} \quad f^{-1}(x) = x - 0.8[x]$

$f - g = x + 4[x] - x + 0.8[x] = \boxed{4.8[x]}$