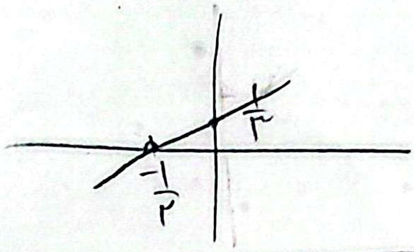
نوید لطیفی — کلاس ۱۰

۱-

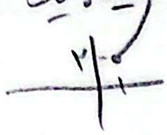
$f(x)=\frac{3}{2}x-\frac{1}{2}$ $\quad f^{-1}(x)=\frac{2}{3}x+\frac{1}{3}$

$+\frac{1}{2}\times\frac{1}{3}\times\frac{1}{2}=\frac{1}{12}$



۲-

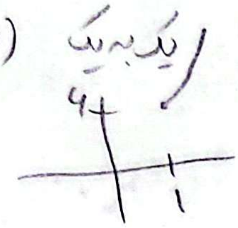
الف) یک به یک



$y=(x-1)^2+2 \to y-2=(x-1)^2 \to \sqrt{y-2}=|x-1|$

$\xrightarrow{x\geq 1} \sqrt{y-2}=x-1 \to x=\sqrt{y-2}+1 \to f^{-1}(x)=\sqrt{x-2}+1$, $x\geq 2$

ب) یک به یک



$y=(x-1)^2+2 \longrightarrow f^{-1}(x)=\sqrt{x-2}+1$, $x\geq 6$

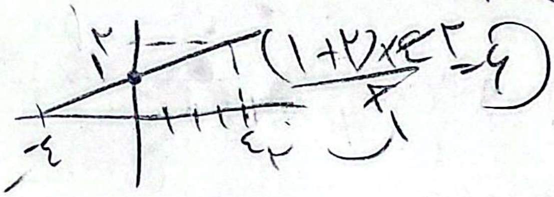
۳-

$f(x)=2x-\sqrt{4x^2-16x+16} \to f(x)=2x-\sqrt{(2x-4)^2}=2x-|2x-4|$ $\begin{cases} x\geq 2 & +4 \times \\ x\leq 2 \to 4x-4 & \checkmark \end{cases}$

$f(x)=4x-4 \quad x\leq 2$

$f^{-1}(x)=\frac{x+4}{4} \quad x\leq 4$

$\frac{x}{4}+1$



۴-

$g_2=g_1 \quad \frac{x_1}{\sqrt{x_1^2-5}}=\frac{x_2}{\sqrt{x_2^2-5}} \to \frac{x_1^2}{x_1^2-5}=\frac{x_2^2}{x_2^2-5} \to x_1^2x_2^2-5x_1^2=x_1^2x_2^2-5x_2^2$

$x_1^2=x_2^2 \to x_1=\pm x_2$

$\frac{x_1}{\sqrt{x_1^2-5}}=\frac{x_2}{\sqrt{x_2^2-5}}$ ، هر دو ≥ 0 $\to x_1, x_2$ هم علامت $\to x_1=x_2$

$y^2=\frac{x^2}{x^2-5} \to x^2=\frac{5y^2}{y^2-1} \to |x|=\frac{\sqrt{5}|y|}{\sqrt{y^2-1}} \xrightarrow{\text{x و y هم علامت}} x=\frac{\sqrt{5}y}{\sqrt{y^2-1}} \to f^{-1}(x)=\frac{\sqrt{5}x}{\sqrt{x^2-1}}$

۵-

$x=2 \begin{cases} 4-8+k \\ -4+12+3k \end{cases} \to -4+k \geq -4+12+3k$

$-2k \geq 12$

$k \leq -6$

۶-

$f \begin{cases} 4x & x \geq 0 \\ 2x & x<0 \end{cases} \qquad f^{-1} \begin{cases} \frac{x}{4} & x \geq 0 \\ \frac{x}{2} & <0 \end{cases} \to \frac{\frac{x}{4}+\frac{x}{2}}{2} - \frac{\frac{x}{2}-\frac{x}{4}}{2}|x|$

$f^{-1} = \frac{3}{8}x - \frac{1}{8}|x| \to a \times b = \frac{-3}{64}$

۷-

$f(2) = \frac{6+4}{2+m} = -10 \to 1 = -2-m \to m=-3 \to f(x) = \frac{3x+4}{x-3} \to f = f^{-1}$

$x + f(x) = x + \frac{3x+4}{x-3} = x \to \frac{3x+4}{x-3} = 0 \to x = -\frac{4}{3}$

۸-

$f^{-1}(f(3-2x)) = f^{-1}(2-g(\frac{x}{2})) \to 3-2\alpha = f^{-1}(-2)$

$2-g(\frac{\alpha}{2}) = -2 \to g(\frac{\alpha}{2}) = 4$

$g(\frac{\alpha}{2}) = 4$

$g^{-1}(4) = \frac{\alpha}{2} \to 4g^{-1}(4) = 2\alpha$

$f^{-1}(-2) + 4g^{-1}(4) = 3-2\alpha+2\alpha = 3$

۹-

$y = \frac{5x+3}{1-x} \xrightarrow{\text{قرینه نسبت به مبدأ}} -\left(\frac{-5x+13}{1+x}\right) = \frac{5x+13}{1+x} \xrightarrow{\text{۲ واحد راست}} \frac{5(x-2)+13}{1+(x-2)} = \frac{5x+3}{x-1}$

$\xrightarrow{\text{۳ واحد پایین}} \frac{5x+3}{x-1} - 3 = \frac{5x+3-3x+3}{x-1} = \frac{2x+6}{x-1}$

$f(x) = f^{-1}(x) \to f(x) = x$

$2x+6 = x^2 - x$

$x^2 - 3x - 6 = 0$

$x_1 + x_2 = \frac{-(-3)}{1} = 3$

نوید لطیفی

$y = x + ۴[x] \rightarrow [y] = [x + ۴[x]] = ۵[x] = [y] \rightarrow [x] = \frac{[y]}{۵}$ ۲۰

$x = y - \frac{۴[y]}{۵} \rightarrow f^{-1}(x) = x - \frac{۴[x]}{۵}$

$x + ۴[x] - x + \frac{۴[x]}{۵} = \frac{۲۴[x]}{۵} = ۴/۸[x]$

نوید امینی تکلیف ۱۰