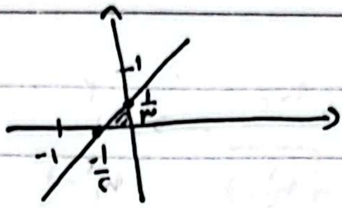
پویا نلفلیان

به نام خدا

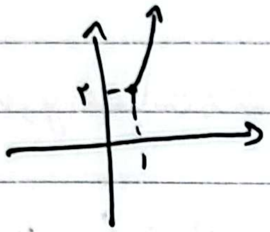
DATE / / SUBJECT:

۱) $\frac{3y-1}{2}=x \to f^{-1}(x)=\frac{2x+1}{3} \to$



$S=\frac{1}{2}\times\frac{1}{2}\times\frac{1}{3}=\boxed{\frac{1}{12}}$

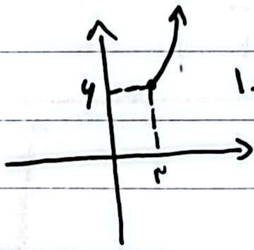
۲) الف)



$\checkmark\ 1-1 \to y=(x-1)^2+2 \to \sqrt{y-2}=|x-1| \to x\geq 1$

$x=\sqrt{y-2}+1 \to f^{-1}(x)=\sqrt{x-2}+1 \;;\; D_{f^{-1}(x)}=[2,+\infty)$

ب) $y=x^2-2x+3$

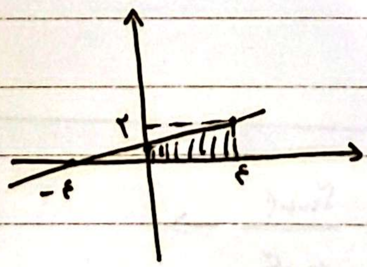


$1-1\checkmark \to y=(x-1)^2+2 \to 3\leq x$

$\sqrt{y-2}+1=x \to f^{-1}(x)=\sqrt{x-2}+1 \;;\; D_{f^{-1}(x)}=[6,+\infty)$

۳) $4x^2-16x+16=4(x^2-4x+4)=4(x-2)^2 \to f(x)=2x-2|x-2| \to$

$$f(x)=\begin{cases}4 & x\geq 2 \quad 1-1 \times\\ 4x-4 & x<2 \quad 1-1\checkmark\end{cases} \to f^{-1}(x)=\frac{x+4}{4} \;;\; D_{f^{-1}(x)}=(-\infty,4]$$



$S=\frac{(1+2)}{2}\times 4=\boxed{6}$

④

(x_1, y)
(x_2, y) $\to \frac{x_1}{\sqrt{x_1^2-5}} = \frac{x_2}{\sqrt{x_2^2-5}} \to \frac{x_1^2}{x_1^2-5} = \frac{x_2^2}{x_2^2-5} \to x_1^2x_2^2-5x_1^2 = x_2^2x_1^2-5x_2^2$

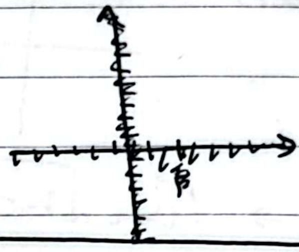
$\to x_1^2 = x_2^2 \to x_1 = \pm x_2 \to$ می‌دانیم x و y هم‌علامت هستند پس ... $x_1 = x_2 \leftarrow$ ۱-۱ است.

$y^2 = \frac{x^2}{x^2-5} \to y^2x^2-5y^2 = x^2 \to x^2 = \frac{5y^2}{y^2-1} \to$

$x = \frac{\sqrt{5}\, y}{\sqrt{y^2-1}} \to \boxed{f^{-1}(x) = \frac{\sqrt{5}\, x}{\sqrt{x^2-1}}}$

⑤

$-4+k \geq 8+3k \to \boxed{k \leq -6}$



⑥

$$f(x) = \begin{cases} 4x & x \geq 0 \\ 3x & x < 0 \end{cases} \to f^{-1}(x) = \begin{cases} \frac{x}{4} & x \geq 0 \\ \frac{x}{3} & x < 0 \end{cases}$$

$\to a+b = \frac{1}{4} , a-b = \frac{1}{3} \to 2a = \frac{7}{12} \to a = \frac{7}{24} , b = \frac{-1}{24} \to$

$\boxed{ab = \frac{-7}{(24)^2}}$

⑦

$f(2) = \frac{10}{2+m} = -10 \to m = -3 \to f(x) = \frac{3x+4}{x-3} \to$

$f^{-1}(x) = \frac{3x+4}{x-3}$, $f \circ f^{-1}(x) = x ; x \in D_{f^{-1}} = R_{f(x)} \to x + \frac{3x+4}{x-3} = x \to$

پویا ملکیان

DATE / / SUBJECT:

ادامه سوال ۷

$x + \frac{3x+4}{x-3} = x \to \frac{3x+4}{x-3} = 0 \to x = \frac{-4}{3} \to$

$R_f = \mathbb{R} - \{3\} \to \boxed{x = \frac{-4}{3}\ \text{ق ق}}$

۸

$f(3-2x) = 2 - g(\frac{x}{2}) = t \to f^{-1}(t) = 3-2x \to x = \frac{3-f^{-1}(t)}{2}$

$\to g^{-1}(2-t) = \frac{x}{2} \to x = 2g^{-1}(2-t)$

$4g^{-1}(2-t) = 3 - f^{-1}(t) \to 4g^{-1}(4) = 3 - f^{-1}(-2) \to 4g^{-1}(4) + f^{-1}(-2) = 3$

۹

قرینه نسبت به مبدأ مختصات $\to y = \frac{+5x+13}{1+x} \to$ ۲ واحد انتقال به راست $\to y = \frac{5x+3}{x-1} \to$

۳ واحد انتقال به پایین $\to y = \frac{5x+3}{x-1} - 3 = \frac{2x+6}{x-1} = f(x) \to$

$\frac{2x+6}{x-1} = x \to x^2 - x = 2x+6 \to x^2 - 3x - 6 = 0 \to$ مجموع طول نقاط $= 3$

۱۰

$y = x + 4[x] \to [y] = 5[x] \to [x] = \frac{[y]}{5} \to$

$y - \frac{4}{5}[y] = x \to f^{-1}(x) = x - \frac{4}{5}[x] \to f(x) - g(x) =$

$x + 4[x] - x + \frac{4}{5}[x] = \frac{24}{5}[x]$