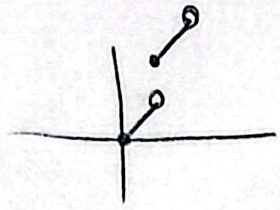


۱۷، ۷۵

توی تستی تلف ۱۱



الف) $0 \leq x < 1 \rightarrow f(x) = x \rightarrow f^{-1}(x) = f(x)$
 ب) $f \circ f^{-1}(x) = 2x - \frac{x}{2} \rightarrow x = 2x - \frac{x}{2} \rightarrow x = \frac{x}{2}$

$$f \circ f^{-1}(x) = 2x - \frac{x}{2} \rightarrow x = 2x - \frac{x}{2} \rightarrow x = \frac{x}{2}$$

جواب ندارد $\rightarrow$ هیچ جوابی ندارد

$$f(x) = \frac{a}{x}$$

$$f^{-1}(x) = -\frac{b}{x} \rightarrow \text{در نقطه } (\frac{a}{x}, \frac{b}{x}) \rightarrow -\frac{b}{x} = \frac{a}{x} \rightarrow a = -b$$

$$\text{if } a = -b \rightarrow f^{-1}(x) = f(x) \rightarrow f \circ f^{-1}(x) = f \circ f(x) = x \rightarrow f \circ f(5-\sqrt{2}) = 5-\sqrt{2}$$

تابع همومورفیسم داشتن خود را روی این خاصه ازمایش کنید

$$\frac{m+n}{n+m+c} = x \rightarrow m+n+c = x^2 + m+n+c \rightarrow x^2 = c$$

$$\frac{1}{a} + \frac{1}{b} = \frac{a+b}{ab} \rightarrow \frac{-\frac{b}{a}}{\frac{a+b}{ab}} = -\frac{b}{a} = \frac{c}{a} \rightarrow c = -b$$

$$f(x) = |x+5| - 1 \text{ for } x \geq -5$$

$$x \geq -5 \rightarrow x+5 = y \rightarrow x = y-5$$

$$-5 \leq x \leq 5 \rightarrow x+5 = y \rightarrow x = y-5$$

$$x \leq -5 \rightarrow x+5 = y \rightarrow x = y-5$$

$$f^{-1}(x) = -x+5$$

$$x \geq -5$$

$$f^{-1}(x) = \frac{-x+5}{2}$$

$$x \leq -5$$

$$Df^{-1} \subset Rf, (-\infty, \frac{29}{2}]$$

②

$$g(n) = y = r \cdot f(rn) - 1 \rightarrow y+1 = f(rn) \rightarrow f^{-1}\left(\frac{y+1}{r}\right) = rn \rightarrow f^{-1}\left(\frac{y+1}{r}\right) = n$$

$$g^{-1}(n) = f^{-1}\left(\frac{n+1}{r}\right) \times \left(\frac{1}{r}\right) \rightarrow a \quad \frac{\frac{1}{r} \times r + 0}{r \times 1} = \frac{r}{r} = 1$$

③

$$f(n) = n + r\sqrt{n} + 1 - 1 = (\sqrt{n+1})^2 - 1 = y \geq 0$$

$$\sqrt{y+1} = \sqrt{n+1} \rightarrow (\sqrt{y+1} - 1)^2 = n \rightarrow f^{-1}(n) = (\sqrt{n+1} - 1)^2 \quad n \geq 0$$

④

$$y = \underbrace{x-1}_{\text{و.ا.}} + \underbrace{x^2}_{\text{و.ا.}} \rightarrow y \text{ bounded } f(n) = f^{-1}(y) \rightarrow f(n) = n$$

$$x-1+x^2 = x \rightarrow x^2 = 1 \rightarrow x = 0 \text{ (نقطه)}$$

⑤

$$f^{-1}(n) \geq n \rightarrow f \circ f^{-1}(n) \geq f(n) \rightarrow n \geq f(n) \rightarrow n^2 + 5n \leq n$$

$$n(n^2 + 5) \leq 0 \rightarrow n \leq 0$$

نتیجه نامطلوب (بازه) $x=0$ است پس $n < 0$ ❌

⑥

$$f(n) \xrightarrow[n=n]{\substack{\text{مقرینه} \\ \text{نقطه} \\ \text{و.ا.}}} f^{-1}(n) \xrightarrow{\text{بازه}} f^{-1}(n+\varepsilon)$$

$$f^{-1}(n+\varepsilon) = n-1$$

$$f \circ f^{-1}(n+\varepsilon) = f(n-1)$$

$$n+\varepsilon = f(n-1) = r - n + \sqrt{n+1} = n+\varepsilon$$

فقط یک مقدار وجود دارد $n=0$ و $n = -\frac{1}{2}$ ❌

⑦

$$f(n) = \sqrt{n+1} \sqrt{y} = r \rightarrow f(n) = f^{-1}(n) \rightarrow f \circ f^{-1}(n) = r$$

$$f^{-1}(n) = \sqrt{y+1} \sqrt{n} = r$$

$n \in \mathbb{Q}_+$

$n \geq 0, \sqrt{y} = r - \sqrt{n} \rightarrow r - \sqrt{n} \geq 0 \rightarrow n \leq r^2 \rightarrow \mathbb{Q}_+ = [0, r^2]$

