



الف) $f(x) = x \rightarrow f^{-1}(x) = f(x)$
 (یعنی متقابل با خودش است)

ب) $f \circ f^{-1}(x) = 2x - \frac{x}{2} \rightarrow x = 2x - \frac{x}{2} \rightarrow x = \frac{3}{2}$

جواب ندارد $\rightarrow$ هیچ جوابی ندارد
 $f^{-1}(\frac{3}{2}) = x \rightarrow f(x) = \frac{3}{2} \rightarrow x = \frac{3}{2}$

الف) $a = \frac{a}{x}$
 ب) $a = -\frac{b}{x} \rightarrow$ معادله $(-\frac{b}{x}, \frac{a}{x})$
 $\frac{a}{x} = -\frac{b}{x} \rightarrow a = -b$

اگر $a = -b \rightarrow f^{-1}(x) = f(x) \rightarrow f \circ f^{-1}(x) = f \circ f^{-1}(x) = x \rightarrow f \circ f^{-1}(a - \sqrt{2}) = a - \sqrt{2}$

الف) $f(x) = |x+1| - |x-1|$
 ب) $x > 1$
 $x+1 - (x-1) = -x+1 \Rightarrow 0$
 $-x+1 + (x-1) = x+1 \Rightarrow 0$
 $x < -1$
 $-x-1 - (x-1) = -x-1 - x+1 = -2x \Rightarrow 0$

$f^{-1}(x) = -x+1$
 $f^{-1}(x) = \frac{-x+1}{2}$

②

$$g(n) = y = r \cdot f(rn) - 1 \rightarrow y+1 = f(rn) \rightarrow f^{-1}\left(\frac{y+1}{r}\right) = rn \rightarrow f^{-1}\left(\frac{y+1}{r}\right) = n$$

$$g^{-1}(n) = f^{-1}\left(\frac{n+1}{r}\right) \times \left(\frac{1}{r}\right) \rightarrow a \quad \frac{\frac{1}{r} \times r + 0}{r \times 1} = \frac{\frac{r}{r}}{\frac{r}{r}} = \frac{1}{r}$$

③

$$f(n) = n + r\sqrt{n} + 1 - 1 = (\sqrt{n+1})^2 - 1 = y \geq 0$$

$$\sqrt{y+1} = \sqrt{n+1} \rightarrow (\sqrt{y+1} - 1)^2 = n \rightarrow f^{-1}(n) = (\sqrt{n+1} - 1)^2 \quad n \geq 0$$

④

$$y = \underbrace{x-1}_{\infty-1} + \underbrace{x^2}_{\infty} \rightarrow y \text{ bounded or } f(n) = f^{-1}(y) \rightarrow f(n) = n$$

$$x-1+x^2 = x \rightarrow x^2 = 1 \rightarrow (x=0) \text{ (نقطه)}$$

⑤

$$f^{-1}(n) \geq n \rightarrow f \circ f^{-1}(n) \geq f(n) \rightarrow n \geq f(n) \rightarrow n^2 + \varepsilon n \leq n$$

$$n(n^2 + \varepsilon) \leq 0 \rightarrow n \leq 0$$

نتیجه نامتنافی (برای) $x=0$ است پس یکی

⑥

$$f(n) \xrightarrow[n=n]{\substack{\text{مقرن} \\ \text{نقطه} \\ \text{نقطه}}} f^{-1}(n) \xrightarrow{\text{نقطه}} f^{-1}(n+\varepsilon)$$

$$f^{-1}(n+\varepsilon) = n - r$$

$$f \circ f^{-1}(n+\varepsilon) = f(n-r)$$

$$n+\varepsilon = f(n-r) = r - n + \sqrt{n+1} = n + \varepsilon$$

فقط نقطه مورد نیاز $n=0$ $n = \frac{1}{r^2} \varepsilon^2 \rightarrow$

⑦

$$f(n) = \sqrt{n+1} \sqrt{y} = r \rightarrow f(n) = f^{-1}(n) \rightarrow f \circ f^{-1}(n) = r$$

$$f^{-1}(n) = \sqrt{y+1} \sqrt{n} = r \quad n \in \mathbb{Q}_+$$

$n \geq 0$, $\sqrt{y} = r - \sqrt{n} \rightarrow r - \sqrt{n} \geq 0 \rightarrow n \leq r^2 \rightarrow \mathbb{Q}_+ = [0, r^2]$

