آوین فتحی‌زاده

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$$\frac{a\sin\frac{4\pi}{3} + b\cos\frac{7\pi}{6}}{a\sin\frac{11\pi}{6} + b\cos\frac{5\pi}{3}} \quad \left(-\sin\frac{\pi}{3},\ -\cos\frac{\pi}{6},\ -\sin\frac{\pi}{6},\ \cos\frac{\pi}{3}\right)$$

$$= \frac{-\frac{\sqrt3}{2}a - \frac{\sqrt3}{2}b}{-\frac{a}{2} + \frac{b}{2}} = \frac{-\sqrt3(a+b)}{-a+b} = \tan\frac{\pi}{6} = \frac{\sqrt3}{3}$$

$$\Rightarrow 3a + 3b = b - a \Rightarrow 4a = -2b \Rightarrow \frac{b}{a} = -2$$

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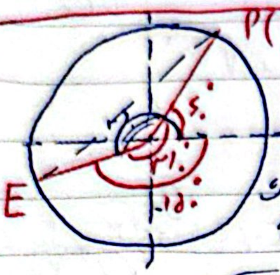

$$P\left(\tfrac{1}{2}, y\right) \Rightarrow \cos\theta = \frac12 \Rightarrow \theta = 60^\circ \Rightarrow 60 - 210 = -150^\circ$$

زاویه شلیک $= 360 - 60 - 150 = 150^\circ \Rightarrow$

$\sin 150^\circ = \sin 30^\circ = \frac12$

$\cos 150^\circ = -\cos 30^\circ = -\frac{\sqrt3}{2}$

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$$P_\Delta = \sqrt{P'^2 + E^2 - 2PE\cos 150} \quad OP + OP' = \left(\sqrt{2+\sqrt3} + 2\right)$$

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ساعت

$$\Rightarrow 360 = 2\pi = 12h \Rightarrow 1h = \frac{2\pi}{12} = \frac{\pi}{6}$$ ⇒ در ساعت $\frac{\pi}{6}$ درجه حرکت کند

$\frac{\pi}{6}$	$1h$
$\frac{\pi}{9}$	$\frac{2}{3}h = 40\,min$

$$\Rightarrow 20:00 + 0:40 = 20:40\,h$$

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فرمول زاویه بین دو عقربه: ← $H = 3,\ \left(\left|\frac{11}{2}M - 30H\right|\right)$

$$\Rightarrow \left|\frac{11}{2}M - 90\right| = 20 \xrightarrow[\text{که قدر مطلق منفی شود پس}]{\text{بار دوم هم بار اول و هنوز آن اتفاق را نیفتد}} \frac{11}{2}M - 90 = 20$$

$$\frac{11}{2}M = 110 \Rightarrow M = 20$$

$$\Rightarrow (3:20)$$

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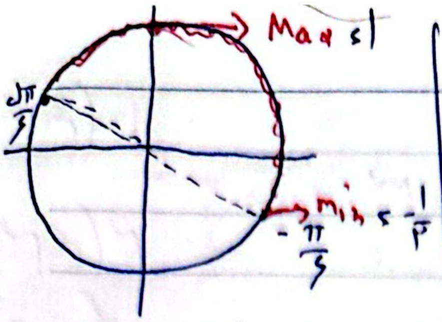

$$\Rightarrow -\frac{1}{2} < \sin\theta \leq 1 \Rightarrow -\frac{1}{2} < \frac{m-3}{5} \leq 1$$

$$\Rightarrow -\frac{5}{2} < m-3 \leq 5 \Rightarrow \frac{1}{2} < m \leq 8$$

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اعداد صحیح $\Leftarrow \{1,2,3,4,5,6,7,8\} \Leftarrow$ ۸ عدد صحیح

۶)

$$\frac{\sin 105 + k\cos 255}{\cos 195 + 2\sin 195} = \frac{\cos 15 - k\sin 15}{-\cos 15 - 2\sin 15}$$

$$\Rightarrow \tan 15 = 0{,}25 \Rightarrow \frac{\sin 15}{\cos 15} = \frac{1}{4} \Rightarrow \sin 15 = 1, \quad \cos 15 = 4$$

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$$\Rightarrow \frac{4-k}{-4-2} = \frac{4-k}{-6} = 3 \Rightarrow 4-k = -18 \Rightarrow -k = -22 \Rightarrow \boxed{k = 22}$$

۷)

$$\frac{1-\cos x}{1+\cos x} = 2 \Rightarrow 1-\cos x = 2+2\cos x \Rightarrow \boxed{\cos x = -\frac{1}{3}}$$

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$\tan x = -\sqrt{8}$

$\boxed{\sin x = \frac{\sqrt{8}}{3}}$ $\quad \cot x = -\frac{1}{\sqrt{8}} = \frac{-1}{2\sqrt{2}}$

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$$\frac{\tan(\frac{\pi}{2}-x) - \cot(\pi - x)}{\cot(\frac{3\pi}{2}-x) + \tan(2\pi - x)} = \frac{2\cot x}{-\tan x} = -2\cot^2 x \qquad \frac{-\sqrt{2}}{4}$$

$$\Rightarrow -2\left(-\frac{\sqrt{2}}{4}\right)^2 = -2 \times \frac{2}{16} = \boxed{-\frac{1}{4}}$$

۸)

$$\frac{108}{180} = \frac{\theta}{\pi} \Rightarrow 108° = \frac{3\pi}{5} \Rightarrow S = \frac{1}{2} r^2 \pi \Rightarrow \frac{S_1}{S_2} = \left(\frac{r_1}{r_2}\right)^2$$

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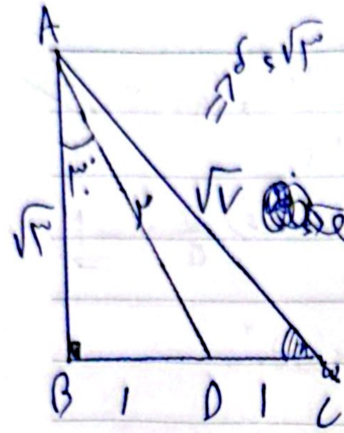
$$\Rightarrow \frac{r_1}{r_2} = \frac{\frac{39\pi}{12}}{\frac{26\pi}{10}} = \frac{3}{2} \Rightarrow \frac{S_1}{S_2} = \left(\frac{3}{2}\right)^2 = \boxed{\frac{9}{4}}$$



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$$\sin 30^\circ = \frac{BD}{AD} = \frac{1}{2} \Rightarrow AD = 2 \Rightarrow \tan 30^\circ = \frac{1}{\sqrt{3}} \Rightarrow AB = \sqrt{3}$$

$$S = \sqrt{3} = \frac{1}{2} BC \times AB = \frac{1}{2} \times BC \times \sqrt{3} = \sqrt{3} \Rightarrow BC = 2$$

$$DC = 1$$

(فیثا) $\Rightarrow \sqrt{BC^2 + AB^2} = AC \Rightarrow \sqrt{4+3} = \sqrt{7}$

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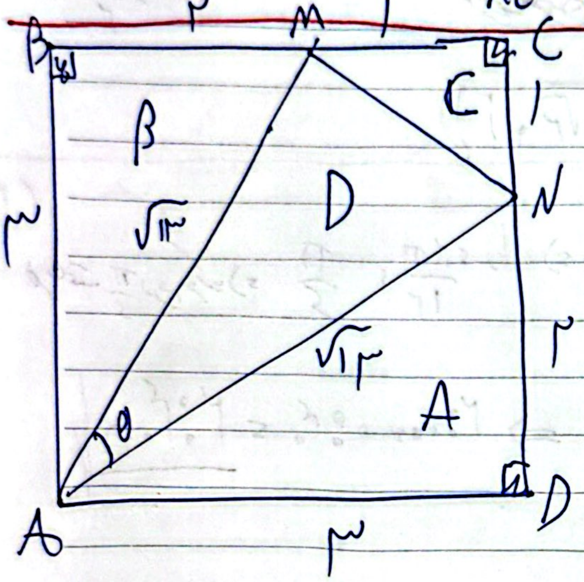
$$\left.\begin{aligned} \sin C &= \frac{AB}{AC} = \frac{\sqrt{3}}{\sqrt{7}} \\ \cos C &= \frac{BC}{AC} = \frac{2}{\sqrt{7}} \end{aligned}\right| \Rightarrow \sin 2C = 2\sin C\cos C = 2 \times \frac{\sqrt{3}}{\sqrt{7}} \times \frac{2}{\sqrt{7}} = \frac{4\sqrt{3}}{7}$$



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$$AM = \sqrt{9+4} = \sqrt{13}$$

$$AN = \sqrt{9+4} = \sqrt{13}$$

$$S_{\square} = 3 \times 3 = 9$$

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$$\left.\begin{aligned} S_A &= \frac{1}{2} \times 2 \times 3 = 3 \\ S_B &= \frac{1}{2} \times 2 \times 3 = 3 \\ S_C &= \frac{1}{2} \times 1 \times 1 = \frac{1}{2} \end{aligned}\right\} S_{A+B+C} = 6 + \frac{1}{2}$$

$$S_D = S_{\square} - S_{A+B+C} = 2\frac{1}{2} = \frac{5}{2}$$

$$S_D = \frac{1}{2} AM\, AN \sin\theta = \frac{5}{2} \Rightarrow \frac{5}{2} = \frac{13}{2} \times \sin\theta \Rightarrow \boxed{\sin\theta = \frac{5}{13}}$$