(۱) $\frac{BC}{\sin\hat{A}} = \frac{AC}{\sin B} = \frac{BC}{\sin(\hat{B}+\hat{C})} \rightarrow \sin\hat{B} = \frac{BC}{BC}\sin(\hat{B}+\hat{C})$

$\sin\hat{B} = \frac{\sqrt{2}}{2}\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{2} \xrightarrow[\hat{B}<120^\circ]{} \hat{B}=45^\circ, \hat{C}=75^\circ$

(۲) $C\hat{A}D = 45^\circ \rightarrow \frac{CD}{\sin 45^\circ} = \frac{AC}{\sin 60^\circ} \Rightarrow AC = 2\sqrt{3}$

$\rightarrow x = AC\sin 60^\circ = 2\sqrt{3}\times\frac{\sqrt{3}}{2} = 3$

(۳) $\frac{b^2-2}{\sin\hat{B}} = \frac{c}{\sin\hat{C}} = \frac{b}{\sin\hat{B}} \rightarrow b^2-b-2 = (b-2)(b+1) = 0 \rightarrow b=2$

(۴) $AC^2 = AB^2 + BC^2 - 2(AB\cdot BC)\cos B$

$\rightarrow 19 = 25 + 4 - 2(2\times 2)\cos B \rightarrow \cos B = 0,8 \rightarrow \sin B = 0,6$

(۵) $a^2 = 4^2 + 4(4+2\sqrt{3}) - 2\times 4\times 2(1+\sqrt{3})\left(\frac{1}{2}\right) = 24 \rightarrow a = 2\sqrt{6}$

$\frac{a}{\sin\hat{A}} = \frac{b}{\sin\hat{B}} \rightarrow \sin\hat{B} = \frac{4}{2\sqrt{6}}\times\frac{\sqrt{3}}{2} = \frac{\sqrt{2}}{2} \xrightarrow[B<120^\circ]{} B=45^\circ, C=75^\circ$

(۶) $ABC: 2^2 = 1^2+2^2 - 2(1\times 2)\cos\alpha \rightarrow 8\cos\alpha = 1$

$ADE \Rightarrow DE^2 = 4^2+6^2 - 2(4\times 6)\left(\frac{1}{8}\right) = 40 \rightarrow DE = 2\sqrt{10}$

(۷) $BC^2 = 5^2+8^2 - 2(5\times 8)\left(\frac{1}{2}\right) = 49 \Rightarrow \underline{\underline{BC=7}}$

$S = \frac{1}{2}ab\sin\hat{A} = \frac{1}{2}(5\times 8)\left(\frac{\sqrt{3}}{2}\right) = 10\sqrt{3}$

(۸ AM

$$(b+c)^2 - a^2 = b^2+c^2+2bc-(b^2+c^2-2bc\cos A)$$
$$= 2bc(1+\cos\hat{A}) \longrightarrow \frac{(b+c)^2-a^2}{bc(1+\cos\hat{A})} = 2$$

(۹

$$a^3+b^3-c^3 = a^3+a^2b-a^2c \to b^3-c^3 = a^2(b-c)$$
$$\longrightarrow a^2 = b^2+c^2+bc = b^2+c^2-2bc\cos\hat{A} \to \cos\hat{A} = -\frac{1}{2}$$

(۱۰ PM

$$S = \frac{1}{2}bc\sin\hat{A} \Rightarrow 2 \longrightarrow bc = 8 = (3-\cos\theta)(1+3\cos\theta)$$
$$= 3+8\cos\theta-3\cos^2\theta$$

$$\longrightarrow -3t^2+8t-5 = 0 = -(t-1)(3t-5) \longrightarrow t=1 \to \cos\theta = 1$$
$$\hookrightarrow t = \frac{5}{3} \times$$

$$-1 \le \cos\theta = -\frac{1}{3}t \le 1$$

$$\longrightarrow c=2, b=4 \longrightarrow a^2 = b^2+c^2-2bc\left(\frac{\sqrt{3}}{2}\right) = 4+16-8(4)\left(\frac{\sqrt{3}}{2}\right)$$
$$= 20-8\sqrt{3}$$