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۱- $\hat B+\hat C=120 \to \hat A=60\ ,\ BC=20$

$\frac{\frac{\sqrt3}{2}=\sin A}{20=BC}=\frac{\sin B}{AC=\frac{20\sqrt6}{3}} \to \sin B=\frac{\cancel{20}\sqrt6\times\sqrt3}{2\times3\times\cancel{20}}=\frac{\sqrt{18}}{6}=\frac{\cancel{3}\sqrt2}{\cancel{6}}=\frac{\sqrt2}{2}\to \boxed{B=45^\circ}$

$B+C=120 \to C=75^\circ$

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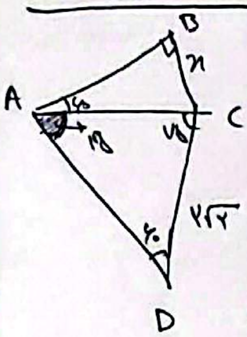
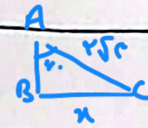
۲- 

$\frac{\frac{\sqrt2}{2}}{2\sqrt2}=\frac{\frac{\sqrt3}{2}}{AC}\to AC=2\sqrt3$



$\to x=\frac{\sqrt3}{2}\times2\sqrt3=3$

$\frac{\sin 45}{2\sqrt2}=\frac{\sin60}{AC}\to\frac{\sqrt2}{2\times\cancel{2}\sqrt2}=\frac{\sqrt3}{\cancel{2}AC}\to\boxed{AC=\frac{2\sqrt3}{\cancel{2}}=\sqrt6}$

$\sin60=\frac{x}{AC}\to\frac{\sqrt3}{2}=\frac{x}{\sqrt6}\to x=\frac{\sqrt{18}}{2}=\frac{3\sqrt2}{2}$

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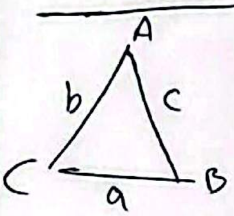
۳- 

$(b^2-2)\frac{\sin C}{\sin B}=c\to(b^2-2)\sin C=c\cdot\sin B\to\frac{\sin B}{b^2-2}=\frac{\sin C}{c}$

$\xrightarrow{\frac{\sin B}{b}=\frac{\sin C}{c}} b^2-2=b\to b^2-b-2=0\to(b-2)(b+1)=0\to b=2$

$\to b=-1$ ×

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۴- $AC^2=AB^2+BC^2-2\times AB\times BC\times\cos B\to169=\underbrace{400+441}_{841}-\underbrace{20\times21\times2\times\cos B}_{-840\cos B}$

$\to 840\cos B=672\to\cos B=\frac{672}{840}=\boxed{\frac45}\xrightarrow{\sin^2+\cos^2=1}\sin^2B+\frac{16}{25}=1\to\sin^2B=\frac{9}{25}$

$\to \sin B=\frac35$

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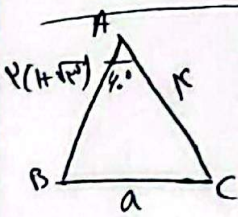
۵- 

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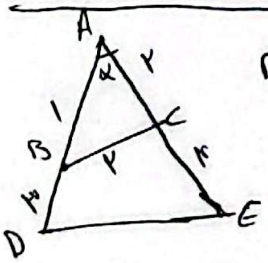
الف) $a=BC=\sqrt{16+16+8\sqrt3-16(1+\sqrt3)\times\frac12}=\sqrt{24}=2\sqrt6$

ب) $\frac{AC}{\sin B}=\frac{BC}{\sin60}\to\frac{4}{\sin B}=\frac{2\sqrt6}{\frac{\sqrt3}{2}}\to\sin B=\frac{\cancel{4}\sqrt3}{\cancel{4}\sqrt6}=\frac{1}{\sqrt2}=\frac{\sqrt2}{2}\to B=45$

$C=180-(60+45)=75^\circ$

۶- 

$BC=2=\sqrt{4+1-4\cos\alpha}\to5-4\cos\alpha=4\to4\cos\alpha=1\to\boxed{\cos\alpha=\frac14}$

$DE=\sqrt{16+36-\frac{48\times1}{\cancel{4}}}=\sqrt{40}=2\sqrt{10}$

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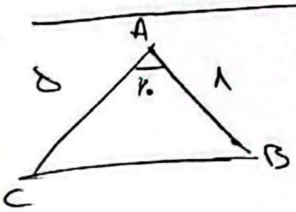
۷- 

الف) $BC=\sqrt{64+25-\underbrace{80\times\frac12}_{-40}}=\sqrt{49}=7$

ب) $S=\frac12 AC\cdot AB\cdot\sin60=\frac{8\times5\times\sqrt3}{\cancel{2\times2}}=10\sqrt3$

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-8

0

$$a^2=b^2+c^2-2bc\cos\hat{A} \longrightarrow \cancel{b^2}+\cancel{c^2}+2bc-\cancel{b^2}-\cancel{c^2}+2bc\cos A$$

$$\frac{2bc(\cos A+1)}{bc(\cos A+1)}=2$$

-9

2

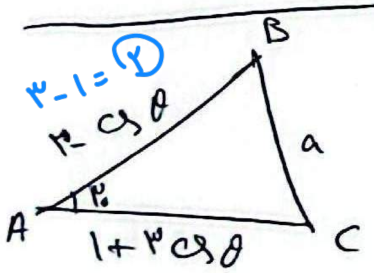
$$\frac{a^3+b^3-c^3}{a+b-c}=a^2 \longrightarrow \cancel{a^3}+b^3-c^3=\cancel{a^3}+a^2b-a^2c \rightarrow \cancel{(b-c)}(b^2+c^2+bc)=a^2\cancel{(b-c)}$$

$$\rightarrow \begin{cases} b^2+c^2+bc=a^2 \\ a^2=b^2+c^2-2bc\cos A \quad \text{از طرفی هم} \end{cases} \Rightarrow \cancel{b^2}+\cancel{c^2}+bc=\cancel{b^2}+\cancel{c^2}-2bc\cos A \rightarrow -2\cancel{bc}\cos=\cancel{bc}$$

$$\rightarrow -2\cos A=1 \rightarrow \cos A=-\frac{1}{2}$$

-10

1.5



$$S=2=\frac{1}{2}\times\frac{1}{2}\times(3\cos\theta)(1+3\cos\theta)$$

$$\rightarrow -9\cos^2\theta+8\cos\theta+3=8 \longrightarrow \frac{-9}{a}\cos^2\theta+\frac{8}{b}\cos\theta-\frac{5}{c}=0$$

$$\rightarrow a+b+c=0 \rightarrow \begin{cases}\cos\theta=1 \checkmark \longrightarrow \boxed{AB=\cancel{3}}\ 2,\ \boxed{AC=4} \\ \cos\theta=\frac{5}{9} \times\end{cases}$$

$$a^2=AB^2+AC^2-2AB.AC.\cos30 \longrightarrow a^2=9+16-24\times\frac{\sqrt{3}}{2}=25-12\sqrt{3}$$

$$a^2=2^2+4^2-2(4)(2)\left(\frac{\sqrt{3}}{2}\right)=20-8\sqrt{3}$$

کامل